CHAPTER 4

MAGNETIC EFFECT OF CURRENT

4.1 CONCEPT OF MAGNETIC FIELD

1. *Briefly explain the concept of magnetic field.*

Concept of magnetic field. A magnet attracts small pieces of iron, cobalt, nickel etc. *The space around a magnet within which its influence can be experienced is called its magnetic field.* However, it is now known that all magnetic phenomena result from forces between electric charges in motion.

In order to explain the *interaction between two charges in motion,* it is useful to introduce the concept of magnetic field, and to describe the interaction in *two* stages :

1. A *moving charge* or a current sets up or creates a *magnetic field* in the space surrounding it.
2. The magnetic field exerts *a force on a moving charge* or a current in the field.

Like electric field, magnetic field is a vector field, that is, a vector associated with each point in space. We use the symbol $\vec{B}$ for a magnetic field.

4.2 OERSTED'S EXPERIMENT

2. *Describe Oersted's experiment leading to the discovery of magnetic effect of current. State Ampere's swimming rule.*

Magnetic effect of current : Historical note. The relation between electricity and magnetism was first noticed by an Italian Jurist. *Gian Demenico Romagnosi* in 1802. He found that an electric current flowing in a wire affects a magnetic needle, and published his observations in a local newspaper, *Gazetta di Trentino.* However, his observations were overlooked. The fact that a magnetic field is associated with an electric current was rediscovered in 1820 by a Danish Physicist, *Hans Christian Oersted.* His observations are explained below.

Oersted's experiment. Consider a magnetic needle *SN* pivoted over a stand. Hold a wire *AB* parallel to the needle *SN* and connect it to a cell and a plug-key, as shown in Fig. 4.1.

It is observed that :

1. When the wire is held above the needle and the current flows from the south to the north, the north pole of the magnetic needle gets deflected towards the west, as shown in Fig. 4.1(*a*).
2. When the direction of the current is reversed, so that it flows from the north to the south, the north pole of the magnetic needle gets deflected towards the east, as shown in Fig. 4.1(*b*).

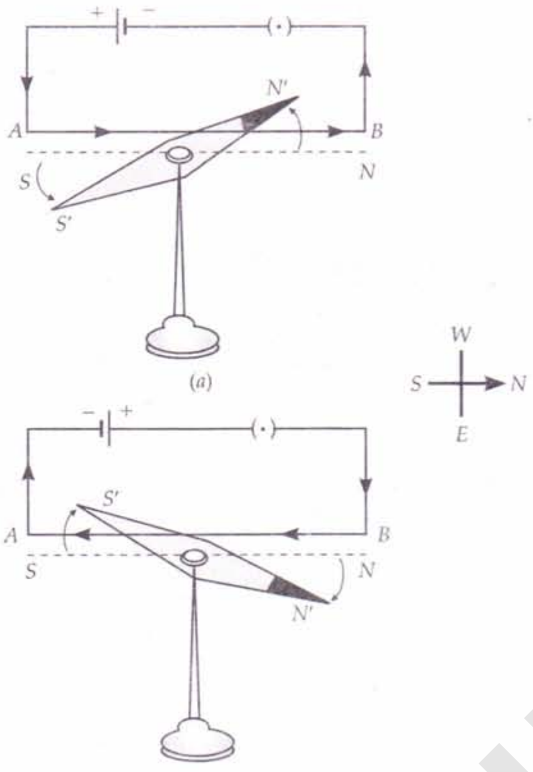


Fig. 4.1 Deflection of a magnetic needle under the influence of electric current.

3. When the wire is placed below the needle, the direction of deflection of the needle is again reversed.
4. When the current in the wire is stopped flowing, the magnetic needle comes back into its initial position.

Since a magnetic needle can be deflected by a magnetic field only, it follows from the above experiment that *a current carrying conductor produces a magnetic field around it.*

Ampere's swimming rule. This rule predicts the direction of deflection of the magnetic needle in the Oersted's experiment, it can be stated as follows :

Imagine a man swimming along the wire in the direction of the flow of the current with his face always turned towards the magnetic needle, then the north pole of the needle will get deflected towards his left hand, as shown in Fig. 4.2.

The direction can also be remembered with the help of the word SNOW. It indicates that if the current flows from **S**outh to **N**orth and the wire is held **O**ver the needle, the north pole is deflected towards the **W**est.

Fig. 4.2 Ampere's swimming rule.

4.3 BIOT-SAVART LAW

3. *State and explain Biot-Savart law for the magnetic field produced by a current element. Define the SI unit of magnetic field from this law.*

Biot-Savart law. Oersted experiment showed that a current carrying conductor produces a magnetic field around it. It is convenient to assume that this field is made of contributions from different segments of the conductor, called *current elements*. A current element is denoted by $\vec{dl}$, which has the same direction as that of current I. From a series of experiments on current carrying conductors of simple shapes, two French physicists *Jean-Baptiste Biot* and *Felix Savart*, in 1820, deduced an expression for the magnetic field of a current element which is known as Biot-Savart law.

Statement. As shown in Fig. 4.3, consider a current element $\vec{dl}$ of a conductor XY carrying current I. Let P be the point where the magnetic field $\vec{dB}$ due to the current element $\vec{dl}$ is to be calculated. Let the position vector of point P relative to element $\vec{dl}$ be $\vec{r}$. Let θ be the angle between $\vec{dl}$ and $\vec{r}$.

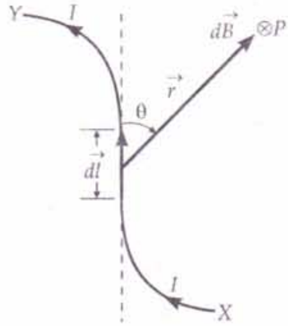


Fig. 4.3 Biot-Savart law.

According to ***Biot-Savart law***, the magnitude of the field $\vec{dB}$ is

1. *directly proportional to the current I through the conductor,*

$$dB \propto I$$

2. *directly proportional to the length dl of the current element,*

$$dB \propto dl$$

3. *directly proportional to sin θ,*

$$dB \propto \sin\theta$$

4. *inversely proportional to the square of the distance r of the point P from the current element,*

$$dB \propto \frac{1}{r^2}$$

Combining all these four factors, we get

$$dB \propto \frac{I\,dl \sin\theta}{r^2}$$

or
$$dB = K \cdot \frac{I\,dl \sin\theta}{r^2}$$

The proportionality constant K depends on the medium between the observation point P and the current element and the system of units chosen. For free space and in SI units,

$$K = \frac{\mu_0}{4\pi} = 10^{-7}\ \text{T m A}^{-1}\ (\text{or Wb m}^{-1}\text{A}^{-1})$$

Here μ_0 is a constant called *permeability* of free space. So the Biot-Savart law in SI units may be expressed as

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I\,dl \sin\theta}{r^2}$$

We can write the above equation as

$$dB = \frac{\mu_0}{4\pi} \frac{I\,dl\,r \sin\theta}{r^3}$$

As the direction of $\vec{dB}$ is perpendicular to the plane of $\vec{dl}$ and $\vec{r}$, so from the above equation, we get the *vector form of the Biot-Savart law* as

$$\vec{dB} = \frac{\mu_0 I}{4\pi} \frac{\vec{dl} \times \vec{r}}{r^3}$$

Direction of $\vec{dB}$. The direction of $\vec{dB}$ is the direction of the vector $\vec{dl} \times \vec{r}$. It is given by *right hand screw rule.* If we place a right handed screw at point P perpendicular to the plane of paper and turn its handle from $\vec{dl}$ to $\vec{r}$, then the direction in which the screw advances gives the direction $\vec{dB}$. Thus the direction of $\vec{dB}$ is *perpendicular to* and *into* the plane of paper, as has been shown by encircled cross ⊗ at point P in Fig. 4.3.

Special Cases

1. If $\theta = 0°$, $\sin\theta = 0$, so that $dB = 0$
   i.e., the magnetic field is zero at points on the axis of the current element.
2. If $\theta = 90°$, $\sin\theta = 1$, so that dB is maximum *i.e., the magnetic field due to a current element is maximum in a plane passing through the element and perpendicular to its axis.*

SI unit of magnetic field from Biot-Savart law. The SI unit of magnetic field is *tesla*, named after the great Yugoslav inventor and scientist *Nikola Tesla*. According to Biot-Savart law,

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I\,dl \sin\theta}{r^2}$$

If $I = 1\,\text{A}, dl = 1\,\text{m}, r = 1\,\text{m}$ and $\theta = 90°$ so that $\sin\theta = 1$, then

$$dB = \frac{\mu_0}{4\pi}$$

$$= \frac{4\pi \times 10^{-7}}{4\pi} = 10^{-7}\ \text{tesla}$$

Thus **one tesla** *is 10^7 times the magnetic field produced by a conducting wire of length one metre and carrying current of one ampere at a distance of one metre from it and perpendicular to it.*

4.4 BIOT-SAVART LAW VS. COULOMB'S LAW

4. *Give some points of similarities and differences between Biot-Savart law for the magnetic field and Coulomb's law for the electrostatic field.*

Comparison of Biot-Savart law with Coulomb's law. According to ***Coulomb's law***, the electric field produced by a charged element at a distance r is given by

$$dE = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r^2}$$

According to ***Biot-Savart*** law, the magnetic field produced by a current element $I\,\vec{dl}$ at a distance r is given by

$$dB = \frac{\mu_0}{4\pi} \frac{I\,dl \sin\theta}{r^2}$$

On comparing the above two equations, we note the following points of similarities and differences between the two laws.

Points of similarity :

1. Both fields depend inversely on the square of the distance from the source to the point of observation.
2. Both are long range fields.
3. The principle of superposition is applicable to both fields. This is because the magnetic field is linearly related to its source, namely, the current element $I\,\vec{dl}$ and the electrostatic field is related linearly to its source, namely, the electric charge.

Points of difference :

1. The magnetic field is produced by a vector source : the current element $I\,\vec{dl}$. The electrostatic field is produced by a scalar source : the electric charge dq.
2. The direction of the electrostatic field is along the displacement vector joining the source and the field point. The direction of the magnetic field is perpendicular to the plane containing the displacement vector $\vec{r}$ and the current element $I\,\vec{dl}$.
3. In Bio-Savart law, the magnitude of the magnetic field is proportional to the sine of the angle between the current element $I\,\vec{dl}$ and displacement vector $\vec{r}$ while there is no such angle dependence in the Coulomb's law for the electrostatic field. Along the axial line of the current element $\theta = 0°$, $\sin\theta = 0$ and hence $dB = 0$.

5. *Write a relation between μ_0, ε_0 and c.*

Relation between μ_0, ε_0 and c. We know that

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ Nm}^2 \text{ C}^{-2}$$

and $$\frac{\mu_0}{4\pi} = 10^{-7} \text{ Tm A}^{-1}$$

$$\therefore \quad \mu_0 \varepsilon_0 = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{4\pi\varepsilon_0}{1}\right)$$

$$= 10^{-7} \times \frac{1}{9 \times 10^9} = \frac{1}{(3 \times 10^8)^2}$$

But $3 \times 10^8 \text{ ms}^{-1}$ = speed of light in vacuum (c)

$$\therefore \quad \mu_0 \varepsilon_0 = \frac{1}{c^2}$$

or $$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}.$$

Examples Based on Biot-Savart Law

Formula Used

Biot-Savart law, $dB = \frac{\mu_0}{4\pi} \frac{Idl \sin\theta}{r^2}$

Units Used

Magnetic field B is in tesla, current I in ampere and distance r in metre.

Constant Used

Permeability constant, $\mu_0 = 4\pi \times 10^{-7}$ Tm A^{-1}

Example 1. *A wire placed along the north-south direction carries a current of 8 A from south to north. Find the magnetic field due to a 1 cm piece of wire at a point 200 cm north-east from the piece.*

Solution. The problem is illustrated in Fig. 4.4.

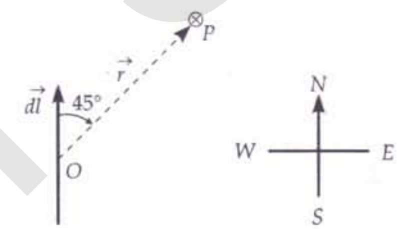


Fig. 4.4

As the distance OP is much larger than the length of the wire, we can treat the wire as a small current element.

Here $I = 8$ A, $dl = 1 \text{ cm} = 1 \times 10^{-2}$ m,
$r = 200 \text{ cm} = 2$ m, $\theta = 45°$

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I\,dl \sin\theta}{r^2}$$

$$= \frac{4\pi \times 10^{-7}}{4\pi} \cdot \frac{8 \times 1 \times 10^{-2} \times \sin 45°}{2^2}$$

$$= \mathbf{1.4 \times 10^{-9}\ T}.$$

The direction of the magnetic field at point P is normally into the plane of paper.

Example 2. *An element $\Delta\vec{l} = \Delta x\,\hat{i}$ is placed at the origin and carries a large current $I = 10$ A. What is the magnetic field on the y-axis at a distance of 0.5 m. $\Delta x = 1$ cm* [NCERT]

Solution. Here $dl = \Delta x = 1 \text{ cm} = 10^{-2}$m, $I = 10$ A, $r = y = 0.5$ m, $\theta = 90°$, $\mu_0 / 4\pi = 10^{-7}$ Tm A^{-1}

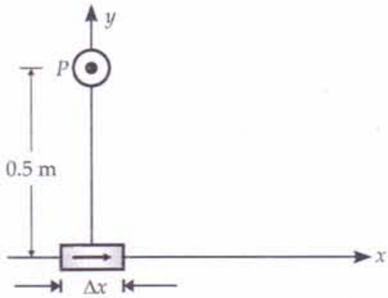


Fig. 4.5

According to Biot-Savart law,

$$dB = \frac{\mu_0}{4\pi}\frac{Idl \sin\theta}{r^2}$$

$$= \frac{10^{-7} \times 10 \times 10^{-2} \times \sin 90°}{(0.5)^2}$$

$$= \mathbf{4 \times 10^{-8}\ T}$$

The direction of the field $\vec{dB}$ will be the direction of vector $\vec{dl} \times \vec{r}$. But

$$\vec{dl} \times \vec{r} = \Delta x\,\hat{i} \times y\,\hat{j} = \Delta x\, y\,(\hat{i} \times \hat{j}) = \Delta x\, y\,\hat{k}$$

Hence field $\vec{dB}$ is in the + z-direction.

Problems for Practice

1. A wire placed along east-west direction carries a current of 10 A from west to east direction. Determine the magnetic field due to a 1.8 cm piece of wire at a point 300 cm north-east from the piece.
(Ans. 1.4×10^{-9} T, normally out of the plane of paper)

2. A small current element $I\,\vec{dl}$, with $\vec{dl} = 2\,\hat{k}$ mm and $I = 2$ A is centred at the origin. Find magnetic field $\vec{dB}$ at the following points :
   (*i*) On the x-axis at $x = 3$ m.
   (Ans. $4.44 \times 10^{-11}\,\hat{j}$ T)
   (*ii*) On the x-axis at $x = -6$ m.
   (Ans. $-1.11 \times 10^{-11}\,\hat{j}$ T)
   (*iii*) On the z-axis at $z = 3$ m. (Ans. 0)

3. An element $\vec{\Delta l} = \Delta x\,\hat{i}$ is placed at the origin (as shown in Fig. 4.6) and carries a current $I = 2$ A. Find out the magnetic field at a point P on the y-axis at a distance of 1.0 m due to the element $\Delta x = 1$ cm. Give also the direction of the field produced. **[CBSE D 09C]**
(Ans. 2×10^{-9}T, in + z-direction)

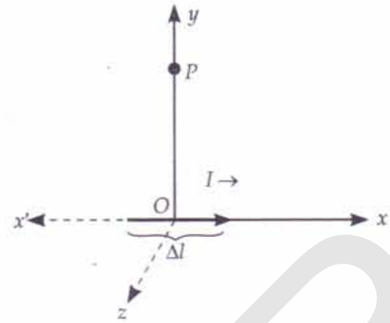


Fig. 4.6

HINTS

1. Proceed as in Example 1.
2. Proceed as in Example 2.
3. Proceed as in Example 2.

We shall now apply Biot-Savart law to calculate the magnetic field due to (*i*) a straight current carrying conductor and (*ii*) a circular current loop.

4.5 MAGNETIC FIELD DUE TO A LONG STRAIGHT CURRENT CARRYING CONDUCTOR

6. *Apply Biot-Savart law to derive an expression for the magnetic field produced at a point due to the current flowing through a straight wire of infinite length. Also draw the sketch of the magnetic field. State the rules used for finding the direction of this magnetic field.*

Magnetic field due to a long straight current carrying conductor. As shown in Fig. 4.7, consider a straight conductor XY carrying current I. We wish to find its magnetic field at the point P whose perpendicular distance from the wire is a *i.e.*, $PQ = a$.

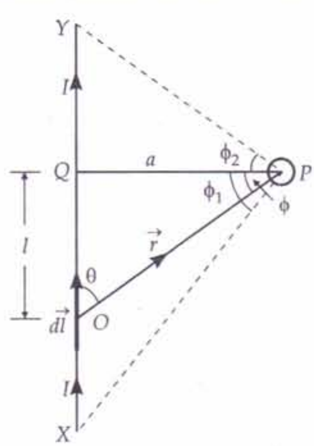


Fig. 4.7 Magnetic field due to a straight current carrying conductor.

Consider a small current element $\vec{dl}$ of the conductor at O. Its distance from Q is l *i.e.*, $OQ = l$. Let $\vec{r}$ be the position vector of point P relative to the current element and θ be the angle between $\vec{dl}$ and $\vec{r}$. According to Biot-Savart law, the magnitude of the field $\vec{dB}$ due to the current element $\vec{dl}$ will be

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I\,dl \sin\theta}{r^2}$$

From right $\Delta\, OQP$,

$$\theta + \phi = 90°$$

or $$\theta = 90° - \phi$$

$\therefore$ $$\sin\theta = \sin(90° - \phi) = \cos\phi$$

Also $$\cos\phi = \frac{a}{r}$$

or $$r = \frac{a}{\cos\phi} = a \sec\phi$$

As $$\tan\phi = \frac{l}{a}$$

$\therefore$ $$l = a\tan\phi$$

On differentiating, we get

$$dl = a\sec^2\phi\, d\phi$$

Hence $$dB = \frac{\mu_0}{4\pi} \frac{I(a\sec^2\phi\, d\phi)\cos\phi}{a^2\sec^2\phi}$$

or $$dB = \frac{\mu_0 I}{4\pi a}\cos\phi\, d\phi$$

According to right hand rule, the direction of the magnetic field at the P due to all such current elements will be in the same direction, namely, normally into the plane of paper. Hence the total field $\vec{B}$ at the point P due to the entire conductor is obtained by integrating the above equation within the limits $-\phi_1$ and ϕ_2.

$$B = \int_{-\phi_1}^{\phi_2} dB = \frac{\mu_0 I}{4\pi a}\int_{-\phi_1}^{\phi_2} \cos\phi\, d\phi$$

$$= \frac{\mu_0 I}{4\pi a}[\sin\phi]_{-\phi_1}^{\phi_2}$$

$$= \frac{\mu_0 I}{4\pi a}[\sin\phi_2 - \sin(-\phi_1)]$$

or $$B = \frac{\mu_0 I}{4\pi a}[\sin\phi_1 + \sin\phi_2]$$

This equation gives magnetic field due to a finite wire in terms of the angles subtended at the observation point by the ends of the wire.

Special Cases

1. *If the conductor XY is infinitely long and the point P lies near the middle of the conductor,* then $\phi_1 = \phi_2 = \pi/2$.

$\therefore$ $$B = \frac{\mu_0 I}{4\pi a}[\sin 90° + \sin 90°]$$

or $$B = \frac{\mu_0 I}{2\pi a}$$

2. *If the conductor XY is infinitely long but the point P lies near the end Y (or X),* then $\phi_1 = 90°$ and $\phi_2 = 0°$.

$\therefore$ $$B = \frac{\mu_0 I}{4\pi a}[\sin 90° + \sin 0°] = \frac{\mu_0 I}{4\pi a}.$$

Clearly, the magnetic field due to an infinitely long straight current carrying conductor at its one end is just half of that at any point near its middle, provided the two points are at the same perpendicular distance from the conductor.

3. *If the conductor is of finite length L and the point P lies on its perpendicular bisector,* then $\phi_1 = \phi_2 = \phi$ and

$$\sin\phi = \frac{L/2}{\sqrt{a^2 + (L/2)^2}} = \frac{L}{\sqrt{4a^2 + L^2}}$$

$\therefore$ $$B = \frac{\mu_0 I}{4\pi a}[\sin\phi + \sin\phi]$$

$$= \frac{\mu_0 I}{4\pi a} \cdot \frac{2L}{\sqrt{4a^2 + L^2}}$$

or $$B = \frac{\mu_0 IL}{2\pi a\sqrt{4a^2 + L^2}}.$$

Direction of magnetic field. For an infinitely long conductor,

$$B = \frac{\mu_0 I}{2\pi a}$$

i.e., $$B \propto \frac{1}{a}$$

Clearly, the magnitude of the magnetic field will be same at all points located at the same distance from the conductor. Hence *the magnetic lines of force of a straight*

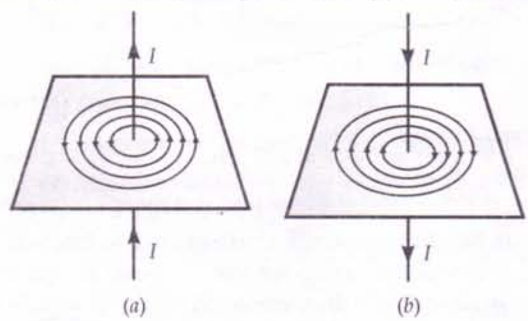


Fig. 4.8 Magnetic lines of foce of a straight current carrying conductor.

current carrying conductor are *concentric circles with the wire at the centre and in a plane perpendicular to the wire.* [A line of force is a curve, the tangent to which at any point gives the direction of magnetic field at that point]. If the current flows upwards, the lines of force have anticlockwise sense [Fig. 4.8(*a*)] and if the current flows downwards, then the lines of force have clockwise sense [Fig. 4.8(*b*)].

Rules for finding the direction of magnetic field due to straight current carrying conductor. Either of the following two rules can be used for this purpose :

1. ***Right hand thumb rule.*** *If we hold the straight conductor in the grip of our right hand in such a way that the extended thumb points in the direction of current, then the direction of the curl of the fingers will give the direction of the magnetic field* (Fig. 4.9).

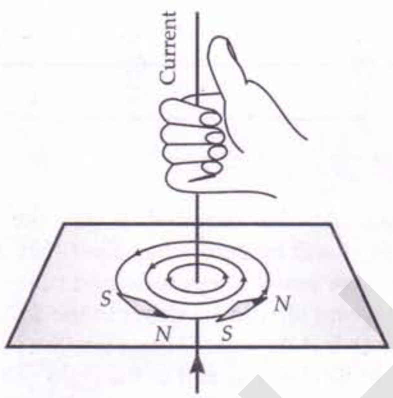


Fig. 4.9 Right hand rule for field due to a straight conductor

2. ***Maxwell's cork screw rule.*** *If a right handed screw be rotated along the wire so that it advances in the direction of current, then the direction in which the thumb rotates gives the direction of the magnetic field* (Fig. 4.10).

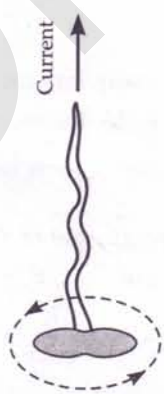


Fig. 4.10 Cork screw rule for field due to a straight conductor.

Variation of magnetic field with distance from straight current carrying conductor. For a straight current carrying conductor,

$$B \propto \frac{1}{a}$$

Thus the graph plotted between the magnetic field B and the distance a from the straight conductor is a hyperbola, as shown in Fig. 4.11.

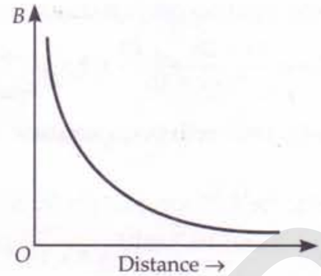


Fig. 4.11 Variation of B with distance from a straight conductor.

Examples based on
Magnetic Field due to Straight Current Carrying Conductor

Formulae Used

1. Magnetic field due to a straight conductor of finite length,

$$B = \frac{\mu_0 I}{4\pi a}(\sin \phi_1 + \sin \phi_2)$$

2. Magnetic field due to an infinitely long straight conductor,

$$B = \frac{\mu_0 I}{2\pi a}$$

Units Used

Magnetic field B is in tesla, current I in ampere and distance a in metre.

Example 3. *A current of 10 A is flowing east to west in a long wire kept horizontally in the east-west direction. Find magnetic field in a horizontal plane at a distance of*

(*i*) *10 cm north*

(*ii*) *20 cm south from the wire ;*

and in the vertical plane at a distance of

(*iii*) *40 cm downward and*

(*iv*) *50 cm upward.*

Solution. (*i*) Magnetic field in a horizontal plane at 10 cm north of the wire is

$$B_N = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 0.10} = \mathbf{2 \times 10^{-5}\ T}$$

According to right hand thumb rule, the direction of the magnetic field will be downward in the vertical plane.

(*ii*) Magnetic field at 20 cm south of the wire is

$$B_S = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 0.20} = \mathbf{1 \times 10^{-5}\ T}$$

The magnetic field will point upward in the vertical plane.

(*iii*) Magnetic field 40 cm just down the wire is

$$B_D = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 0.40} = \mathbf{5 \times 10^{-6}\ T}$$

The magnetic field will point south in a horizontal plane.

(*iv*) Magnetic field 50 cm just above the wire is

$$B_U = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 0.50} = \mathbf{4 \times 10^{-6}\ T}$$

The magnetic field will point north in a horizontal plane.

Example 4. *A long straight wire carrying a current of 30 A is placed in an external uniform magnetic field of 4.0×10^{-4} T parallel to the current. Find the magnitude of the resultant magnetic field at a point 2.0 cm away from the wire.*

Solution. Here $I = 30$ A, $r = 2.0$ cm $= 2.0 \times 10^{-2}$ m

Field due to straight current carrying wire is

$$B_1 = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 30}{2\pi \times 2.0 \times 10^{-2}} = 3.0 \times 10^{-4}\ \text{T}$$

This field will act perpendicular to the external field $B_2 = 4.0 \times 10^{-4}$T. Hence the magnitude of the resultant field is

$$B = \sqrt{B_1^2 + B_2^2} = \sqrt{(3 \times 10^{-4})^2 + (4.0 \times 10^{-4})^2}$$

$$= \mathbf{5 \times 10^{-4}\ T.}$$

Example 5. *Figure 4.12 shows two current-carrying wires 1 and 2. Find the magnitudes and directions of the magnetic field at points P, Q and R.*

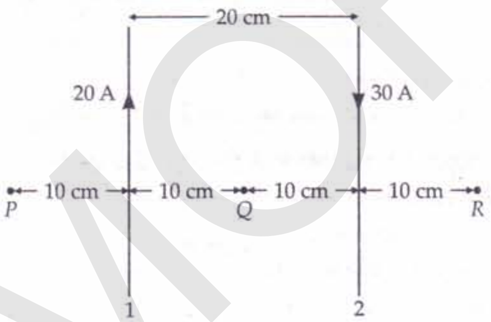


Fig. 4.12

Solution. (*i*) According to right hand grip rule, the field B_1 of wire 1 at point P will point normally outward while the field B_2 of wire 2 will point normally inward, hence

$$B_P = B_1 - B_2 = \frac{\mu_0 I_1}{2\pi r_1} - \frac{\mu_0 I_2}{2\pi r_2}$$

$$= \frac{4\pi \times 10^{-7}}{2\pi}\left[\frac{20}{0.10} - \frac{30}{0.30}\right]$$

$= \mathbf{2 \times 10^{-5}\ T}$, pointing normally outward.

(*ii*) At point Q, both B_1 and B_2 will point normally inward,

$$\therefore\ B_Q = B_1 + B_2 = \frac{4\pi \times 10^{-7}}{2\pi}\left[\frac{20}{0.10} + \frac{30}{0.10}\right]$$

$= \mathbf{10^{-4}\ T}$, pointing normally inward.

(*iii*) At point R, B_1 points normally inward and B_2 points normally outward,

$$\therefore\ B_R = B_2 - B_1 = \frac{4\pi \times 10^{-7}}{2\pi}\left[\frac{30}{0.10} - \frac{20}{0.30}\right]$$

$= \mathbf{4.5 \times 10^{-5}\ T}$, pointing normally outward.

Example 6. *Two parallel wires P and Q placed at a separation of $r = 6$ cm carry electric currents $I_1 = 5$ A and $I_2 = 2$ A in opposite directions as shown in Fig. 4.13. Find the point on the line PQ where the resultant magnetic field is zero.*

Fig. 4.13

Solution. At the required point, the resultant magnetic field will be zero when the fields due to the two wires have equal magnitude and opposite directions. Such point should lie either to the left of P or to the right of Q. But the wire Q has a smaller current, the point should lie closer to and to the right of Q. Let this point be R at distance x from Q, as shown in Fig. 4.13.

Field due to current I_1 at point R,

$$B_1 = \frac{\mu_0 I_1}{2\pi(r + x)},$$

normally into the plane of paper.

Field due to current I_2 at point R,

$$B_2 = \frac{\mu_0 I_2}{2\pi x},$$

normally out of the plane of plane

But $B_1 = B_2$

$$\therefore\ \frac{I_1}{r + x} = \frac{I_2}{x}$$

or $$x = \frac{I_2 r}{I_1 - I_2}$$

$$= \frac{2\ \text{A} \times 6\ \text{cm}}{5\ \text{A} - 2\ \text{A}} = \mathbf{4\ cm.}$$

Example 7. *Use Biot-Savart law to obtain an expression for the magnetic field at the centre of a coil bent in the form of a square of side 2a carrying current I.*

Solution. Refer to Fig. 4.14. Magnetic field at O due to finite length of wire AB is

$$B_1 = \frac{\mu_0 I}{4\pi a}(\sin\alpha + \sin\beta)$$

$$= \frac{\mu_0 I}{4\pi a}(\sin 45^\circ + \sin 45^\circ) = \frac{\sqrt{2}\,\mu_0 I}{4\pi a}$$

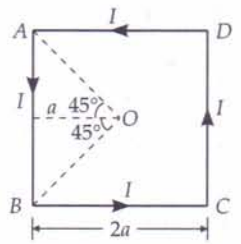


Fig. 4.14

The magnetic field at O due to conductors BC, CD and DA will also be of same magnitude and direction. Therefore, resultant field at O is

$$B = 4\,B_1 = \frac{4\times\sqrt{2}\,\mu_0 I}{4\pi a} = \frac{\sqrt{2}\,\mu_0 I}{\pi a},$$

directed normally outwards.

Example 8. *A current of 1.0 A is flowing in the sides of an equilateral triangle of side 4.5×10^{-2} m. Find the magnetic field at the centroid of the triangle.* [Roorkee 91]

Solution. The situation is shown in Fig. 4.15. The magnetic field at the centre O due to the current through side PQ is given by

$$B_1 = \frac{\mu_0 I}{4\pi a}[\sin\theta_1 + \sin\theta_2]$$

where a is the distance of PQ from O and θ_1, θ_2 are the angles as shown. The magnetic field due to each of the three sides is the same in magnitude and direction, therefore, total magnetic field at O is

$$B = 3\,B_1 = \frac{3\mu_0 I}{4\pi a}\quad[\sin\theta_1 + \sin\theta_2]$$

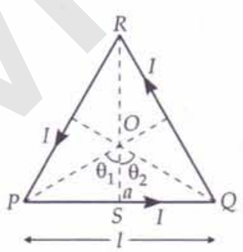


Fig. 4.15

Here $I = 1.0$ A, $\theta_1 = \theta_2 = 60^\circ$, $\mu_0 = 4\pi\times10^{-7}\,\text{Tm A}^{-1}$

$$\frac{PS}{OS} = \tan\theta_1 \quad\text{or}\quad \frac{l/2}{a} = \tan 60^\circ$$

$$\therefore\quad a = \frac{l}{2\tan 60^\circ} = \frac{4.5\times10^{-2}}{2\sqrt{3}}\,\text{m}$$

$$\therefore\quad B = \frac{3\times4\pi\times10^{-7}\times1.0\times2\sqrt{3}}{4\pi\times4.5\times10^{-2}}[\sin 60^\circ + \sin 60^\circ]$$

$$= \frac{6\sqrt{3}\times10^{-5}}{4.5}\left[\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right] = \mathbf{4\times10^{-5}\ T},$$

directed normally outwards.

Example 9. *Figure 4.16 shows a right-angled isosceles ΔPQR having its base equal to a. A current of I ampere is passing downwards along a thin straight wire cutting the plane of paper normally as shown at Q. Likewise a similar wire carries an equal current passing normally upwards at R. Find the magnitude and direction of the magnetic induction B at P. Assume the wires to be infinitely long.* [ISCE 97]

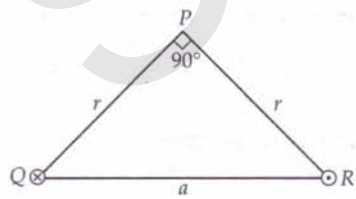


Fig. 4.16

Solution. Let $PQ = QR = r$. In right ΔPQR,

$$a^2 = r^2 + r^2 = 2r^2 \qquad\text{or}\qquad r = \frac{a}{\sqrt{2}}$$

Magnetic induction at point P due to the conductor passing through Q,

$$B_1 = \frac{\mu_0 I}{2\pi r} = \frac{\sqrt{2}\,\mu_0 I}{2\pi a} = \frac{\mu_0 I}{\sqrt{2}\,\pi a},\ \text{acting along } PR$$

Magnetic induction at point P due to the conductor passing through R,

$$B_2 = \frac{\mu_0 I}{\sqrt{2}\,\pi a},\ \text{acting along } PQ$$

As the two fields at point P are acting along perpendicular directions, the resultant magnetic induction at point P is

$$B = \sqrt{B_1^2 + B_2^2}$$

$$= \sqrt{\left(\frac{\mu_0 I}{\sqrt{2}\,\pi a}\right)^2 + \left(\frac{\mu_0 I}{\sqrt{2}\,\pi a}\right)^2} = \sqrt{2}\cdot\frac{\mu_0 I}{\sqrt{2}\,\pi a}$$

or $$B = \frac{\mu_0 I}{\pi a}$$

This field acts towards the midpoint of QR.

Problems for Practice

1. A straight wire carries a current of 3 A. Calculate the magnitude of the magnetic field at a point 10 cm away from the wire. **[CBSE D 96]**
   (Ans. 6×10^{-6} T)
2. At what distance from a long straight wire carrying a current of 12 A will the magnetic field be equal to 3×10^{-5} Wb m^{-2}. (Ans. 8×10^{-2} m)
3. The magnetic induction at a point P which is at a distance of 4 cm from a long current carrying wire is 10^{-3} T. What is the magnetic induction at another point Q which is at a distance of 12 cm from this current carrying wire ? (Ans. 3.33×10^{-4} T)
4. What current must flow in an infinitely long straight wire to give a flux density of 3×10^{-5} T at 6 cm from the wire ? (Ans. 9 A)
5. A vertical wire in which a current is flowing produces a neutral point with the earth's magnetic field at a distance of 10 cm from the wire. What is the current if $B_H = 1.8 \times 10^{-4}$ T ? (Ans. 90 A)
6. Fig. 4.17 shows two long, straight wires carrying electric currents of 10 A each in opposite directions. The separation between the wires is 5.0 cm. Find the magnetic field at a point P midway between the wires.

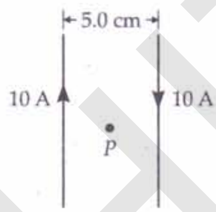


Fig. 4.17

   (Ans. 1.6×10^{-6} T)
7. Two long parallel wires are placed at a distance of 16 cm from each other in air. Each wire has a current of 4 A. Calculate the magnetic field at midpoint between them when the currents in them are (*i*) in the same direction and (*ii*) in opposite directions. [Ans. (*i*) Zero (*ii*) 2×10^{-5} T]
8. Two infinitely long insulated wires are kept perpendicular to each other. They carry currents $I_1 = 2$ A and $I_2 = 1.5$ A. (*i*) Find the magnitude and direction of the magnetic field at P. (*ii*) If the direction of current is reversed in one of the wires, what would be the magnitude of the field B ?
   [Ans. (*i*) 2×10^{-5} T, normally into the plane of paper (*ii*) zero]

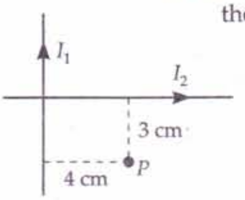


Fig. 4.18

9. A long straight wire carrying a current of 200 A, runs through a cubical box, entering and leaving through holes in the centres of opposite faces, as shown in Fig. 4.19. Each side of the box is of 20 cm.

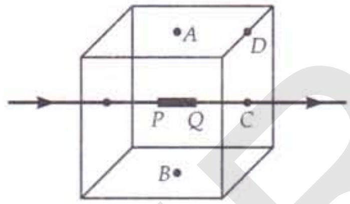


Fig. 4.19

   Consider an element PQ of the wire 1 cm long at the centre of the box. Calculate the magnetic field produced by this element at the points A, B, C and D. The points A, B and C are the centres of the faces of the cube and D is the midpoint of one edge.
   (Ans. 20×10^{-6} T, 20×10^{-6} T, 0, 7.07×10^{-6} T)
10. A long straight telephone cable contains six wires, each carrying a current of 0.5 A. The distance between the wires is negligible. What is the magnitude of magnetic field at a distance of 10 cm from the cable (*i*) if the currents in all the six wires are in same direction (*ii*) if four wires carry current in one direction and the other two in opposite direction. [Ans. (*i*) 6.0×10^{-6}T, (*ii*) 2.0×10^{-6} T]
11. Calculate the magnetic induction at the centre of a coil bent in the form of a square of side 10 cm carrying a current of 10 A. **[Punjab 01]**
    (Ans. 1.13×10^{-4} T)
12. A closed circuit is in the form of a regular hexagon of side a. If the circuit carries current I, what is magnetic induction at the centre of the hexagon ? **[IPUEE 13]**
    $\left(\text{Ans. } B = \dfrac{\sqrt{3}\,\mu_0 I}{\pi a}\right)$
13. Two straight long conductors AOB and COD are perpendicular to each other and carry currents I_1 and I_2 respectively. Find the magnitude of the magnetic field at a point P at a distance a from the point O in a direction perpendicular to the plane $ABCD$.
    $\left(\text{Ans. } \dfrac{\mu_0}{2\pi a}(I_1^2 + I_2^2)^{1/2}\right)$
14. Two insulating infinitely long conductors carrying currents I_1 and I_2 lie mutually perpendicular to each other in the same plane, as shown in Fig. 4.20. Find the magnetic field at the point $P(a, b)$.
    $\left(\text{Ans. } \dfrac{\mu_0}{2\pi}\left(\dfrac{I_2}{b} - \dfrac{I_1}{a}\right), \text{directed inward}\right)$

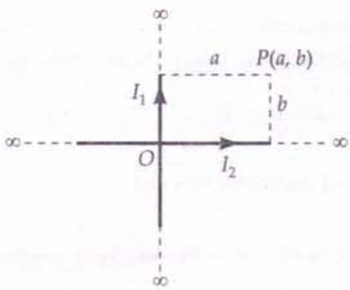


Fig. 4.20

HINTS

1. $B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 3}{2\pi \times 0.10} = \mathbf{6.0 \times 10^{-6}\ T}$.

2. $r = \frac{\mu_0 I}{2\pi B} = \frac{4\pi \times 10^{-7} \times 12}{2\pi \times 3 \times 10^{-5}} = \mathbf{8 \times 10^{-2}\ m}$.

3. Magnetic field due to a straight current carrying conductor,

$$B = \frac{\mu_0 I}{2\pi r} \quad i.e., \quad B \propto \frac{1}{r}$$

$$\therefore \quad \frac{B_Q}{B_P} = \frac{r_P}{r_Q}$$

or $$B_Q = \frac{r_P}{r_Q} . B_P = \frac{4}{12} \times 10^{-3} = \mathbf{3.33 \times 10^{-4}\ T.}$$

4. $I = \frac{2\pi r B}{\mu_0} = \frac{2\pi \times 6 \times 10^{-2} \times 3 \times 10^{-5}}{4\pi \times 10^{-7}} = \mathbf{9\ A.}$

5. If neutral point is obtained at distance r from the wire, then

$$\frac{\mu_0 I}{2\pi r} = B_H$$

or $$I = \frac{2\pi r B_H}{\mu_0} = \frac{2\pi \times 0.10 \times 1.8 \times 10^{-4}}{4\pi \times 10^{-7}} = \mathbf{90\ A.}$$

6. According to right hand thumb rule, the direction of magnetic field due to current in each wire is perpendicular to and pointing into the plane of paper. Hence total field at point P is

$$B = 2 \times \frac{\mu_0 I}{2\pi r}$$

$$= \frac{2 \times 4\pi \times 10^{-7} \times 10}{2\pi \times 2.5 \times 10^{-2}} = \mathbf{1.6 \times 10^{-6}\ T.}$$

$$\left[r = \frac{5}{2}\ \text{cm} = 2.5 \times 10^{-2}\ \text{m} \right]$$

7. (*i*) When the currents are in same direction,

$$B = B_1 - B_2$$

(*ii*) When the currents are in opposite directions,

$$B = B_1 + B_2.$$

8. (*i*) $B_1 = \frac{\mu_0 I_1}{2\pi r_1} = \frac{4\pi \times 10^{-7} \times 2}{2\pi \times 4 \times 10^{-2}} = 10^{-5}\ \text{T}$,

normally into the plane of paper.

$B_2 = \frac{\mu_0 I_2}{2\pi r_2} = \frac{4\pi \times 10^{-7} \times 1.5}{2\pi \times 3 \times 10^{-2}} = 10^{-5}\ \text{T}$,

normally into the plane of paper

$\therefore \quad B = B_1 + B_2 = \mathbf{2 \times 10^{-5}\ T,}$

normally into the plane of paper.

(*ii*) When current in any one wire is reversed, the two fields will be in opposite directions, so that B = **zero.**

9. Here $I = 200$ A, $PQ = dl = 1$ cm $= 0.01$ m

For point A or B, $r = 10$ cm $= 0.1$ m, $\theta = 90°$, therefore

$$B_A = B_B = \frac{\mu_0}{4\pi} \frac{I\,dl \sin\theta}{r^2}$$

$$= \frac{10^{-7} \times 200 \times 0.01 \times \sin 90°}{(0.1)^2} = \mathbf{20 \times 10^{-6}\ T}$$

For point C, $\theta = 0°$, therefore

$$B_C = \frac{\mu_0}{4\pi} . \frac{I\,dl \sin 0°}{r^2} = \mathbf{0.}$$

For point D,

$$r = \sqrt{10^2 + 10^2} = 10\sqrt{2}\ \text{cm} = 0.1\sqrt{2}\ \text{m},\ \theta = 45°$$

$$B_D = \frac{\mu_0}{4\pi} . \frac{I\,dl \sin 45°}{r^2} = \frac{10^{-7} \times 200 \times 0.01 \times 1}{(0.1\sqrt{2})^2 \times \sqrt{2}}$$

$$= \mathbf{7.07 \times 10^{-6}\ T.}$$

10. (*i*) Net current, $I = 0.5 \times 6 = 3.0$ A, $r = 10$ cm $= 0.1$ m

$$B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 3.0}{2\pi \times 0.1} = \mathbf{6.0 \times 10^{-6}\ T.}$$

(*ii*) Net current, $I = 0.5 \times 4 - 0.5 \times 2 = 1.0$ A

$$B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 1.0}{2\pi \times 0.1} = \mathbf{2.0 \times 10^{-6}\ T.}$$

11. Refer to Fig. 4.21. Magnetic field at O due to finite wire AB,

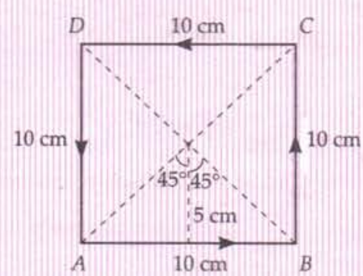


Fig. 4.21

$$B_1 = \frac{\mu_0 I}{4\pi a}(\sin\alpha + \sin\beta)$$

$$= \frac{4\pi \times 10^{-7} \times 10}{4\pi \times 0.05}(\sin 45° + \sin 45°)$$

$$= 2.83 \times 10^{-5}\,\text{T}$$

Total magnetic induction at O,

$$B = 4B_1 = 4 \times 2.83 \times 10^{-5} = \mathbf{1.13 \times 10^{-4}\,T},$$

directed normally outward.

12. $ON = a' = \sqrt{a^2 - \frac{a^2}{4}} = \frac{\sqrt{3}a}{2}$

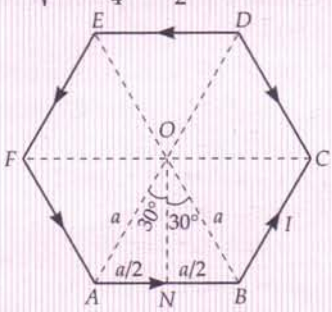


Fig. 4.22

Magnetic field at O due to current in AB is

$$B_1 = \frac{\mu_0 I}{4\pi a'}[\sin\alpha + \sin\beta]$$

$$= \frac{\mu_0 I}{4\pi\sqrt{3}\,a/2}[\sin 30° + \sin 30°] = \frac{\mu_0 I}{2\pi\sqrt{3}a}$$

Total field at $O = 6\,B_1 = \dfrac{\sqrt{3}\mu_0 I}{\pi a}$.

13. Magnetic field at P due to current in wire AOB,

$$B_1 = \frac{\mu_0 I_1}{2\pi a}$$

Magnetic field at P due to current in wire COD,

$$B_2 = \frac{\mu_0 I_2}{2\pi a}$$

As the two conductors are perpendicular to each other, so B_1 and B_2 will also be perpendicular to each other. Hence the resultant magnetic field at P is

$$B = \sqrt{B_1^2 + B_2^2} = \left[\left(\frac{\mu_0 I_1}{2\pi a}\right)^2 + \left(\frac{\mu_0 I_2}{2\pi a}\right)^2\right]^{1/2}$$

$$= \frac{\mu_0}{2\pi a}(I_1^2 + I_2^2)^{1/2}.$$

14. Magnetic field at point P due to current I_1,

$$B_1 = \frac{\mu_0 I_1}{2\pi a}, \text{ directed normally inward}$$

Magnetic field at point P due to current I_2,

$$B_2 = \frac{\mu_0 I_2}{2\pi b}, \text{ directed normally outward}$$

As $b < a$, so $B_2 > B_1$

Hence the net magnetic field at the point P,

$$B = B_2 - B_1 = \frac{\mu_0}{2\pi}\left(\frac{I_2}{b} - \frac{I_1}{a}\right),$$

directed normally inward.

4.6 MAGNETIC FIELD AT THE CENTRE OF CIRCULAR CURRENT LOOP

7. *Apply Biot-Savart law to derive an expression for the magnetic field at the centre of a current carrying circular loop.*

Magnetic field at the centre of a circular current loop. As shown in Fig. 4.23, consider a circular loop of wire of radius r carrying current I. We wish to calculate its magnetic field at the centre O. The entire loop can be divided into a large number of small current elements.

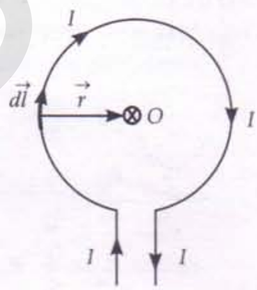


Fig. 4.23 Magnetic field at the centre of a circular current loop.

Consider a current element $\vec{dl}$ of the loop. According to Biot-Savart law, the magnetic field at the centre O due to this element is

$$\vec{dB} = \frac{\mu_0 I}{4\pi}\cdot\frac{\vec{dl} \times \vec{r}}{r^3}$$

The field at point O points normally into the plane of paper, as shown by encircled cross ⊗. The direction of $\vec{dl}$ is along the tangent, so $\vec{dl} \perp \vec{r}$. Consequently, the magnetic field at the centre O due to this current element is

$$dB = \frac{\mu_0 I}{4\pi}\frac{dl \sin 90°}{r^2} = \frac{\mu_0 I}{4\pi}\cdot\frac{dl}{r^2}$$

The magnetic field due to all such current elements will point into the plane of paper at centre O. Hence the total magnetic field at the centre O is

$$B = \int dB = \int \frac{\mu_0 I}{4\pi}\cdot\frac{dl}{r^2} = \frac{\mu_0 I}{4\pi r^2}\int dl$$

$$= \frac{\mu_0 I}{4\pi r^2}\cdot l = \frac{\mu_0 I}{4\pi r^2}\cdot 2\pi r$$

or

$$B = \frac{\mu_0 I}{2r}$$

If instead of a single loop, there is a coil of N turns, all wound over one another, then

$$B = \frac{\mu_0 N I}{2a}.$$

4.7 MAGNETIC FIELD ON THE AXIS OF A CIRCULAR CURRENT LOOP

8. *Apply Biot-Savart law to find the magnetic field due to a circular current carrying loop at a point on the axis of the loop. State the rules used to find the direction of this magnetic field.*

Magnetic field along the axis of a circular current loop. Consider a circular loop of wire of radius a and carrying current I, as shown in Fig. 4.24. Let the plane of the loop be perpendicular to the plane of paper. We wish to find field $\vec{B}$ at an axial point P at a distance r from the centre C.

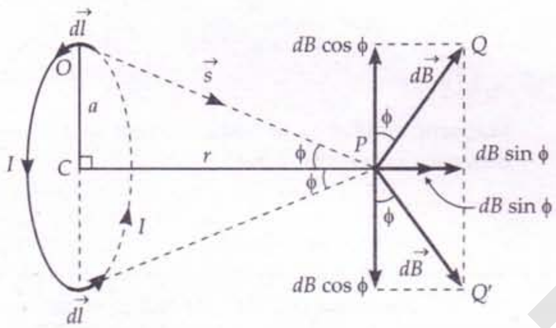


Fig. 4.24 Magnetic field on the axis of a circular current loop.

Consider a current element $\vec{dl}$ at the top of the loop. It has an outward coming current.

If $\vec{s}$ be the position vector of point P relative to the element $\vec{dl}$, then from Biot-Savart law, the field at point P due to the current element is

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I\,dl \sin\theta}{s^2}$$

Since $\vec{dl} \perp \vec{s}$, *i.e.*, $\theta = 90°$, therefore

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I\,dl}{s^2}$$

The field $d\vec{B}$ lies in the plane of paper and is perpendicular to $\vec{s}$, as shown by $\vec{PQ}$. Let ϕ be the angle between OP and CP. Then dB can be resolved into *two* rectangular components.

1. $dB \sin\phi$ along the axis,
2. $dB \cos\phi$ perpendicular to the axis.

For any two diametrically opposite elements of the loop, the components perpendicular to the axis of the loop will be equal and opposite and will cancel out. Their axial components will be in the same direction, *i.e.*, along CP and get added up.

$\therefore$ Total magnetic field at the point P in the direction CP is

$$B = \int dB \sin\phi$$

But $\quad \sin\phi = \frac{a}{s} \quad$ and $\quad dB = \frac{\mu_0}{4\pi} \cdot \frac{I\,dl}{s^2}$

$$\therefore \qquad B = \int \frac{\mu_0}{4\pi} \cdot \frac{I\,dl}{s^2} \cdot \frac{a}{s}$$

Since μ_0 and I are constant, and s and a are same for all points on the circular loop, we have

$$B = \frac{\mu_0 Ia}{4\pi s^3} \int dl = \frac{\mu_0 Ia}{4\pi s^3} \cdot 2\pi a = \frac{\mu_0 Ia^2}{2s^3}$$

$$\left[\because \int dl = \text{circumference} = 2\pi a\right]$$

or $$B = \frac{\mu_0 Ia^2}{2(r^2 + a^2)^{3/2}} \qquad [\because s = (r^2 + a^2)^{1/2}]$$

As the direction of the field is along +ve X-direction, so we can write

$$\vec{B} = \frac{\mu_0 Ia^2}{2(r^2 + a^2)^{3/2}} \hat{i}$$

If the coil consists of N turns, then

$$B = \frac{\mu_0 N Ia^2}{2(r^2 + a^2)^{3/2}}$$

Special Cases

1. ***At the centre of the current loop***, $r = 0$, therefore

$$B = \frac{\mu_0 N Ia^2}{2a^3} = \frac{\mu_0 NI}{2a}$$

or $$B = \frac{\mu_0 N IA}{2\pi a^3}$$

where $A = \pi a^2$ = area of the circular current loop. The field is directed perpendicular to the plane of the current loop.

2. ***At the axial points lying far away from the coil***, $r >> a$, so that

$$B = \frac{\mu_0 N I a^2}{2r^3} = \frac{\mu_0 N IA}{2\pi r^3}$$

This field is directed along the axis of the loop and falls off as the cube of the distance from the current loop.

3. ***At an axial point at a distance equal to the radius of the coil*** *i.e.*, $r = a$, we have

$$B = \frac{\mu_0 N I a^2}{2(a^2 + a^2)^{3/2}} = \frac{\mu_0 NI}{2^{5/2} a}.$$

Direction of the magnetic field. Fig. 4.25 shows the magnetic lines of force of a circular wire carrying current. The lines of force near the wire are almost concentric circles. As we move radially towards the centre of the loop, the concentric circles become larger and larger *i.e.*, the lines of force become less and less curved. If the plane of the circular loop is held perpendicular to the magnetic meridian, the lines at the centre are almost straight, parallel and perpendicular to the plane of the loop. Thus the magnetic field is uniform at the centre of the loop.

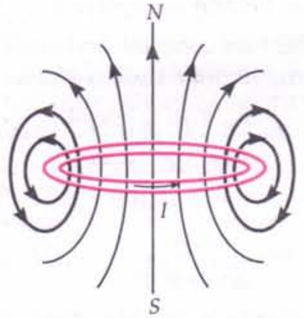


Fig. 4.25 Magnetic lines of force of a circular current loop.

Rules for finding the direction of a magnetic field due to a circular current loop. Either of the following two rules can be used for finding the direction of $\vec{B}$.

1. **Right hand thumb rule.** *If we curl the palm of our right hand around the circular wire with the fingers pointing in the direction of the current, then the extended thumb gives the direction of the magnetic field.*

2. **Clock rule.** *This rule gives the polarity of any face of the circular current loop. If the current round any face of the coil is in anticlockwise direction, it behaves like a north pole. If the current flows in the clockwise direction, it behaves like a south pole* (Fig. 4.26).

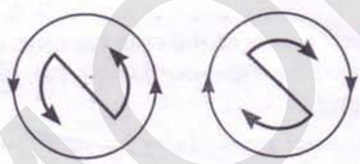


Fig. 4.26 Clock rule.

Variation of the magnetic field along the axis of a circular current loop. Fig. 4.27 shows the variation of the magnetic field along the axis of a circular loop with distance from its centre. The value of B is maximum at the centre, and it decreases as we go away from the centre, on either side of the loop.

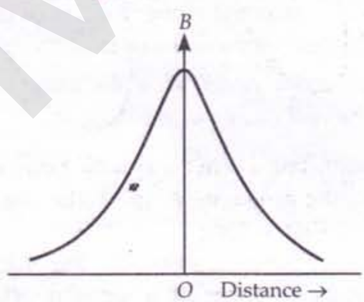


Fig. 4.27 Variation of B along the axis of a circular current loop.

Examples based on

Magnetic Field due to a Circular Coil

Formulae Used

1. Magnetic field at the centre of a circular loop,

$$B = \frac{\mu_0 I}{2r}$$

2. Magnetic field at an axial point of a circular loop,

$$B = \frac{\mu_0 I a^2}{2(r^2 + a^2)^{3/2}}.$$

Units Used

Magnetic field B is in tesla, current in ampere, distances r and a in metre.

Constant Used

$$\mu_0 = 4\pi \times 10^{-7} \text{ Tm A}^{-1}.$$

Example 10. *The plane of a circular coil is horizontal. It has 10 turns each of radius 8 cm. A current of 2 A flows through it. The current appears to flow clockwise from a point above the coil. Find the magnitude and direction of the magnetic field at the centre of the coil due to the current.*

Solution. Here $N = 10$, $r = 8$ cm $= 0.08$ m, $I = 2$ A

$$\therefore \quad B = \frac{\mu_0 NI}{2r} = \frac{4\pi \times 10^{-7} \times 10 \times 2}{2 \times 0.08} = \mathbf{1.57 \times 10^{-4}\ T}$$

As the current flows clockwise when seen from above the coil, the magnetic field at the centre of the coil points vertically downwards.

Example 11. *In the Bohr model of hydrogen atom, an electron revolves around the nucleus in a circular orbit of radius 5.11×10^{-11} m at a frequency of 6.8×10^{15} Hz. What is the magnetic field set up at the centre of the orbit ?* [Haryana 97C]

Solution. If n is the frequency of revolution of the electron, then

$$I = ne = 6.8 \times 10^{15} \times 1.6 \times 10^{-19}$$
$$= 6.8 \times 1.6 \times 10^{-4} \text{ A}$$

$$\therefore \quad B = \frac{\mu_0 I}{2r}$$
$$= \frac{4\pi \times 10^{-7} \times 6.8 \times 1.6 \times 10^{-4}}{2 \times 5.11 \times 10^{-11}} = \mathbf{13.4\ T.}$$

Example 12. *The radius of the first orbit of hydrogen atom is 0.5 Å. The electron moves in an orbit with a uniform speed of $2.2 \times 10^6\ ms^{-1}$. What is the magnetic field* produced at the centre of the nucleus due to the motion of this electron ? Use $\mu_0 / 4\pi = 10^{-7}\ NA^{-2}$ and electronic charge $= 1.6 \times 10^{-19}$ C. [ISCE 98]

Solution. Here $r = 0.5\ \text{Å} = 0.5 \times 10^{-10}$ m,

$$v = 2.2 \times 10^6\ \text{ms}^{-1}$$

Period of revolution of electron,

$$T = \frac{2\pi r}{v} = \frac{2 \times 22 \times 0.5 \times 10^{-10}}{7 \times 2.2 \times 10^6} = \frac{1}{7} \times 10^{-15}\ \text{s}$$

Equivalent current,

$$I = \frac{\text{Charge}}{\text{Time}} = \frac{e}{T} = \frac{1.6 \times 10^{-19} \times 7}{10^{-15}} = 1.12 \times 10^{-3}\ \text{A}$$

Magnetic field produced at the centre of the nucleus,

$$B = \frac{\mu_0 I}{2r} = \frac{4\pi \times 10^{-7} \times 1.12 \times 10^{-3}}{2 \times 0.5 \times 10^{-10}} = \mathbf{14.07\ T.}$$

Example 13. *A helium nucleus is completing one round of a circle of radius 0.8 m in 2 seconds. Show that the magnetic field at the centre of the circle is $10^{-19}\ \mu_0$ tesla. Take $e = 1.6 \times 10^{-19}$ C.*

Solution. The charge on helium nucleus is $+2e$. The revolving nucleus is equivalent to a current-loop.

Current, $I = \dfrac{\text{Charge}}{\text{Time}} = \dfrac{2e}{T}$

Magnetic field at the centre of the circle is

$$B = \frac{\mu_0 I}{2r} = \frac{\mu_0}{2r} \cdot \frac{2e}{T} = \frac{\mu_0 e}{rT}$$

$$= \frac{\mu_0 \times 1.6 \times 10^{-19}}{0.8 \times 2} = \mathbf{10^{-19}\ \mu_0\ tesla.}$$

Example 14. *The magnetic field due to a current-carrying circular loop of radius 12 cm at its centre is 0.50×10^{-4} T. Find the magnetic field due to this loop at a point on the axis at a distance of 5.0 cm from the centre.*

Solution. $B_{centre} = \dfrac{\mu_0 I}{2a}$ and $B_{axial} = \dfrac{\mu_0 I a^2}{2(a^2 + r^2)^{3/2}}$

$$\therefore \quad \frac{B_{axial}}{B_{centre}} = \frac{a^3}{(a^2 + r^2)^{3/2}}$$

or
$$B_{axial} = \frac{a^3}{(a^2 + r^2)^{3/2}} \times B_{centre}$$

Here $a = 12$ cm $= 12 \times 10^{-2}$ m, $r = 5$ cm $= 5 \times 10^{-2}$ m,

$B_{centre} = 0.50 \times 10^{-4}$ T

$$\therefore \quad B_{axial} = \frac{(12 \times 10^{-2})^3}{[144 \times 10^{-4} + 25 \times 10^{-4}]^{3/2}} \times 0.50 \times 10^{-4}\ \text{T}$$

$$= \frac{(12)^3 \times 0.50 \times 10^{-4}}{169 \times 13} = \mathbf{3.9 \times 10^{-5}\ T.}$$

Example 15. *Two identical circular coils of radius 0.1 m, each having 20 turns are mounted co-axially 0.1 m apart. A current of 0.5 A is passed through both of them (i) in the same direction, (ii) in the opposite directions. Find the magnetic field at the centre of each coil.*

Solution. Here $a = 0.1$ m, $N = 20$, $r = 0.1$ m, $I = 0.5$ A

Magnetic field at the centre of each coil due to its own current is

$$B_1 = \frac{\mu_0 NI}{2a} = \frac{4\pi \times 10^{-7} \times 20 \times 0.5}{2 \times 0.1} = 6.28 \times 10^{-5}\ \text{T}$$

Magnetic field at the centre of one coil due to the current in the other coil is

$$B_2 = \frac{\mu_0 NI a^2}{2(a^2 + r^2)^{3/2}}$$

$$= \frac{4\pi \times 10^{-7} \times 20 \times 0.5 \times (0.1)^2}{2[(0.1)^2 + (0.1)^2]^{3/2}} = \frac{0.628 \times 10^{-7}}{[2 \times (0.1)^2]^{3/2}}$$

$$= \frac{0.628 \times 10^{-7}}{2\sqrt{2} \times 10^{-3}} = 2.22 \times 10^{-5}\ \text{T.}$$

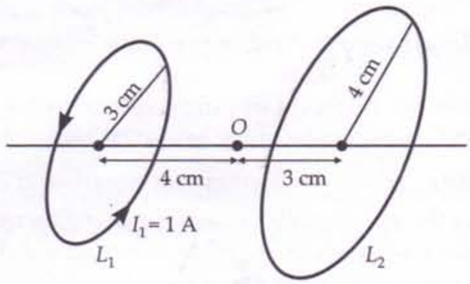
(*i*) When the currents are in the same direction, the resultant field at the centre of each coil is

$$B = B_1 + B_2 = 6.28 \times 10^{-5} + 2.22 \times 10^{-5}$$
$$= \mathbf{8.50 \times 10^{-5}\ T.}$$

(*ii*) When the currents are in opposite directions, the resultant field is

$$B = B_1 - B_2 = 6.28 \times 10^{-5} - 2.22 \times 10^{-5}$$
$$= \mathbf{4.06 \times 10^{-5}\ T.}$$

Example 16. *Two coaxial circular loops L_1 and L_2 of radii 3 cm and 4 cm are placed as shown. What should be the magnitude and direction of the current in the loop L_2 so that the net magnetic field at the point O be zero ?* [CBSE SP 08]



Solution. For the net magnetic field at the point O to be zero, the direction of current in loop L_2 should be opposite to that in loop L_1.

Magnitude of magnetic field due to current I_1 in L_1 = Magnitude of magnetic field due to current I_2 in L_2

or $\dfrac{\mu_0 I_1 (0.03)^2}{2\,[(0.03)^2 + (0.04)^2]^{3/2}} = \dfrac{\mu_0 I_2 (0.04)^2}{2\,[(0.04)^2 + (0.03)^2]^{3/2}}$

or $$I_2 = \frac{(0.03)^2}{(0.04)^2}\, I_1$$

$$= \frac{9}{16} \times 1\ \text{A} = \mathbf{0.56\ A.}$$

Example 17. *A long wire having a semi-circular loop of radius r carries a current I, as shown in Fig. 4.28. Find the magnetic field due to entire wire at the point O.*

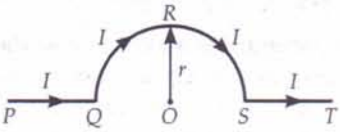


Fig. 4.28

Solution. *Magnetic field due to linear portion.* Any element $\vec{dl}$ *of linear portions like PQ* or *ST* will make angles 0 or π with the position vector $\vec{r}$. Therefore, field at O due to linear portion is

$$B = \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin\theta}{r^2} = 0$$

Magnetic field due to semi-circular portion. Any element $\vec{dl}$ on this portion will be perpendicular to the position vector $\vec{r}$, therefore, field due to one such element at point will be

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin \pi/2}{r^2} = \frac{\mu_0}{4\pi} \frac{Idl}{r^2}$$

Magnetic field due to the entire circular portion is given by

$$B = \int dB = \frac{\mu_0 I}{4\pi r^2} \int dl = \frac{\mu_0 I}{4\pi r^2} \cdot \pi r = \frac{\mu_0 I}{4r}$$

$\therefore$ Total magnetic field at point $O = \dfrac{\mu_0 I}{4r}$.

Example 18. *A straight wire carrying a current of 12 A is bent into a semicircular arc of radius 2.0 cm as shown in Fig. 4.29(a). What is the direction and magnitude of $\vec{B}$ at the centre of the arc ? Would your answer change if the wire were bent into a semicircular arc of the same radius but in the opposite way as shown in Fig. 4.29(b) ?* [NCERT ; Pb 91]

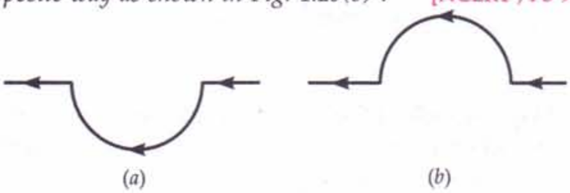


Fig. 4.29

Solution. (*i*) Magnetic field at the centre of the arc is

$$B = \frac{\mu_0 I}{4r}$$

Here $I = 12$ A, $r = 2.0$ cm $= 0.02$ m,

$\mu_0 = 4\pi \times 10^{-7}$ TmA^{-1}

$$\therefore \quad B = \frac{4\pi \times 10^{-7} \times 12}{4 \times 0.02} = \mathbf{1.9 \times 10^{-4}\ T}$$

According to right hand rule, the direction of the field is *normally into the plane of paper.*

(*ii*) The magnetic field will be of same magnitude,

$$B = \mathbf{1.9 \times 10^{-4}\ T}$$

The direction of the field is *normally out of the plane of paper.*

Example 19. *A long wire is bent as shown in Fig. 4.30. What will be the magnitude and direction of the field at the centre O of the circular portion, if a current I is passed through the wire ? Assume that the various portions of the wire do not touch at point P.*

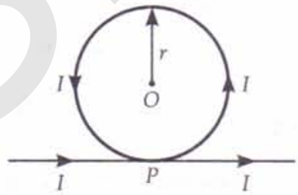


Fig. 4.30

Solution. The system consists of a straight conductor and a circular loop. Field due to straight conductor at point O is

$$B_1 = \frac{\mu_0 I}{2\pi r}, \text{ up the plane of paper}$$

Field due to circular loop at point O is

$$B_2 = \frac{\mu_0 I}{2r}, \text{ up the plane of paper}$$

$\therefore$ Total field at O is

$$B = B_1 + B_2 = \frac{\mu_0 I}{2r}\left(1 + \frac{1}{\pi}\right), \text{ up the plane of paper.}$$

Example 20. *Figure 4.31 shows a current loop having two circular segments and joined by two radial lines. Find the magnetic field at the centre O.*

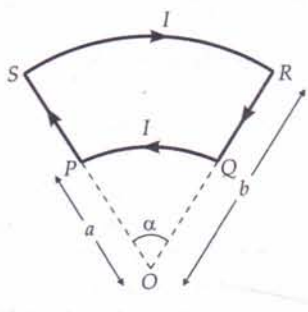


Fig. 4.31

Solution. Since the point O lies on lines SP and QR, so the magnetic field at O due to these straight portions is zero.

The magnetic field at O due to the circular segment PQ is

$$B_1 = \frac{\mu_0}{4\pi}\frac{I}{a^2}\,l$$

Here, l = length of arc $PQ = \alpha\, a$

$$\therefore \quad B_1 = \frac{\mu_0}{4\pi}\frac{I\alpha}{a}, \text{ directed normally upward}$$

Similarly, the magnetic field at O due to the circular segment SR is

$$B_2 = \frac{\mu_0}{4\pi}\cdot\frac{I\alpha}{b}, \text{ directed normally downward.}$$

The resultant field at O is

$$B = B_1 - B_2 = \frac{\mu_0 I\alpha}{4\pi}\left[\frac{1}{a} - \frac{1}{b}\right]$$

or

$$B = \frac{\mu_0 I\,\alpha\,(b-a)}{4\pi\, ab}.$$

Example 21. *The wire shown in Fig. 4.32 carries a current of 10 A. Determine the magnitude of the magnetic field at the centre O. Given radius of the bent coil is 3 cm.* [Punjab 01 ; AIIMS 13]

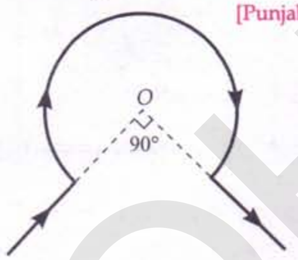


Fig. 4.32

Solution. As $\theta\text{ (rad)} = \dfrac{\text{Arc}}{\text{Radius}}$

$$\therefore \quad \frac{3\pi}{2} = \frac{l}{r} \quad \text{or} \quad l = \frac{3\pi r}{2}$$

According to Biot-Savart law, magnetic field at the centre O is

$$B = \frac{\mu_0}{4\pi}\frac{Il}{r^2} = \frac{\mu_0}{4\pi}\cdot\frac{I}{r^2}\cdot\frac{3\pi r}{2} = \frac{\mu_0}{4\pi}\cdot\frac{3}{2}\cdot\frac{\pi I}{r}$$

$$= \frac{4\pi \times 10^{-7}}{4\pi}\cdot\frac{3}{2}\cdot\frac{22}{7} \times \frac{10}{3 \times 10^{-2}}$$

$$= \mathbf{1.57 \times 10^{-3}\ T.}$$

Example 22. *In Fig. 4.33, abcd is a circular coil of non-insulated thin uniform conductor. Conductors pa and qc are very long straight parallel conductors tangential to the coil at the points a and c. If a current of 5 A enters the coil from p to a, find the magnetic induction at O, the centre of the coil. The diameter of the coil is 10 cm.*

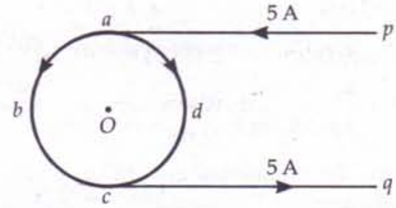


Fig. 4.33

Solution. Here $I_{abc} = I_{adc} = 2.5\text{ A}$,

$$r = Oa = Ob = Oc = Od = 5\text{ cm} = 5 \times 10^{-2}\text{ m.}$$

The magnetic induction at O due to the current in part abc of the coil is equal and opposite to the magnetic induction due to the current in part adc. So magnetic induction at O due to the coil is zero.

Magnetic induction at O due to the straight conductor pa (a half infinite segment) is

$$B_1 = \frac{1}{2}\frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 5}{4\pi \times 5 \times 10^{-2}} = 10^{-5}\text{T},$$

normally out of the plane of paper.

Similarly, magnetic induction at O due to straight conductor qc is

$$B_2 = \frac{\mu_0 I}{4\pi r} = 10^{-5}\text{T},$$

normally out of the plane of paper.

Total magnetic induction at O is

$$B = B_1 + B_2 = 10^{-5} + 10^{-5} = \mathbf{2 \times 10^{-5}\ T},$$

normally out of the plane of paper.

Example 23. *The current-loop PQRSTP formed by two circular segments of radii R_1 and R_2 carries a current of I ampere. Find the magnetic field at the common centre O. What will be the field if angle $\alpha = 90°$?*

Solution. The magnetic field at O due to each of the straight parts PQ and RS is zero because $\theta = 0°$, for each of them.

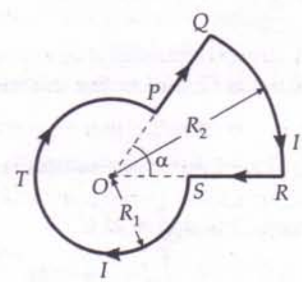


Fig. 4.34

Magnetic field at the centre O due to circular segment QR of radius R_2 is

$$B_1 = \frac{\mu_0}{4\pi}\cdot\frac{I}{R_2^2}\,l_2$$

Here,

l_2 = length of circular segment $QR = \alpha R_2$

$\therefore \quad B_1 = \frac{\mu_0}{4\pi} \cdot \frac{I\alpha}{R_2}$, directed normally downward

Similarly, the magnetic field at O due to the circular segment STP is

$B_2 = \frac{\mu_0}{4\pi} \frac{I(2\pi - \alpha)}{R_1}$, directed normally downward

Hence the resultant field at O is

$$B = B_1 + B_2 = \frac{\mu_0 I}{4\pi}\left(\frac{\alpha}{R_2} + \frac{2\pi - \alpha}{R_1}\right),$$

directed normally downward

If $\qquad \alpha = 90° = \pi/2$, then

$$B = \frac{\mu_0 I}{4\pi}\left(\frac{\pi}{2R_2} + \frac{3\pi}{2R_1}\right) = \frac{\mu_0 I}{8}\left[\frac{1}{R_2} + \frac{3}{R_1}\right].$$

Example 24. *A current $I = 5.0$ A flows along a thin wire shaped as shown in Fig. 4.35. The radius of the curved part of the wire is equal to $R = 120$ mm, the angle $2\phi = 90°$. Find the magnetic induction of the field at the point O.*

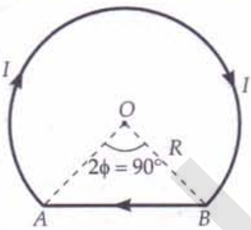


Fig. 4.35

Solution. Magnetic induction at O due to the line segment AB is

$$B_1 = \frac{\mu_0}{4\pi} \times \frac{I}{R\cos\phi}[\sin\phi + \sin\phi]$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{2I}{R}\tan\phi, \text{ acting normally downwards}$$

Magnetic field at O due to the current through arc segment is

$B_2 = \frac{\mu_0}{4\pi} \times \frac{I}{R}(2\pi - 2\phi)$, acting normally downwards

Total magnetic induction at O,

$$B = B_1 + B_2 = \frac{\mu_0}{2\pi} \cdot \frac{I}{R}[\pi - \phi + \tan\phi]$$

$$= \frac{4\pi \times 10^{-7} \times 5}{2\pi \times 0.120}\left[\pi - \frac{\pi}{4} + \tan\frac{\pi}{4}\right]$$

$$= \frac{2 \times 10^{-7} \times 5 \times 3.356}{0.120} = \mathbf{2.8 \times 10^{-5}\ T.}$$

Example 25. *Two wires A and B have the same length equal to 44 cm and carry a current of 10 A each. Wire A is bent into a circle and wire B into a square. (a) Which wire produces a greater magnetic field at the centre ? (b) Obtain the magnitudes of the fields at the centres of the two wires.*

Solution. Given $I = 10$ A,

Length of each wire = 44 cm = $4L$ (say)

(*a*) Suppose the wire is bent into a circle of radius R. Then its perimeter $2\pi R = 4L$

$\therefore$ Magnetic field at the centre of the circular wire is

$$B = \frac{\mu_0 I}{2R} = \frac{\mu_0 I\pi}{2\pi R} = \frac{\mu_0 I\pi}{4L} \qquad ...(1)$$

Now suppose the wire B is bent into a square of side L. We know that the magnetic field due to a wire of finite length whose ends make angles α and β with the perpendicular dropped on wire from the given point at distance r from it is given by

$$dB = \frac{\mu_0 I}{4\pi r}(\sin\alpha + \sin\beta)$$

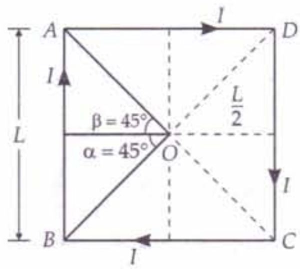


Fig. 4.36

$\therefore$ Magnetic field at O due to conductor AB is

$$dB = \frac{\mu_0 I}{4\pi . L/2}(\sin 45° + \sin 45°) = \frac{2\sqrt{2}\,\mu_0 I}{4\pi L}$$

$[\because\ \alpha = \beta = 45°, r = L/2]$

By symmetry, magnetic field at O due to all the four sides of the square will be in the same direction. Hence total field at O due to the current-carrying square is

$$B = 4 \times \frac{2\sqrt{2}\,\mu_0 I}{4\pi L} = \frac{8\sqrt{2}\mu_0 I}{4\pi L} \qquad ...(2)$$

Comparing equations (1) and (2), we find that the square wire produces a greater field at its centre.

(*b*) Magnetic field at the centre of the circular wire is

$$B = \frac{\mu_0 I\pi}{4L} = \frac{4\pi \times 10^{-7} \times 10 \times \pi}{44 \times 10^{-2}}\ \text{T}$$

$$= \mathbf{0.9 \times 10^{-4}\ T} \qquad [\because\ 4L = 44\ \text{cm}]$$

Magnetic field at the centre of the square wire is

$$B=\frac{8\sqrt{2}\times\mu_0 I}{4\pi L}=\frac{8\times1.414\times4\pi\times10^{-7}\times10}{\pi\times44\times10^{-2}}\text{ T}$$

$$\simeq \mathbf{1.0\times10^{-4}\ T.}$$

Example 26. *A straight wire, of length $\frac{\pi}{2}$ metre, is bent into a circular shape. If the wire were to carry a current of 5 A, calculate the magnetic field, due to it, before bending, at a point distant 0.01 times the radius of the circle formed from it. Also calculate the magnetic field, at the centre of the circular loop formed, for the same value of current.*

[CBSE OD 04C]

Solution. Here $2\pi r=\frac{\pi}{2}$ metre

$$r=\frac{1}{4}=0.25\text{ m}$$

Magnetic field due to straight wire,

$$B=\frac{\mu_0 I}{2\pi r'}=\frac{\mu_0 I}{2\pi\times0.01\,r}=\frac{4\pi\times10^{-7}\times5}{2\pi\times0.01\times0.25}$$

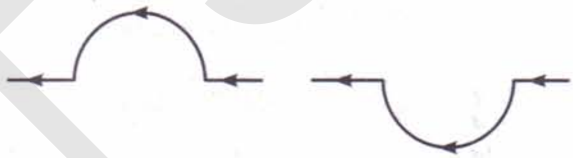
$$=\mathbf{4\times10^{-4}\ T}$$

Magnetic field at the centre of the circular loop,

$$B=\frac{\mu_0 I}{2r}=\frac{4\pi\times10^{-7}\times5}{2\times0.25}=\mathbf{1.256\times10^{-5}\ T.}$$

Problems for Practice

1. Consider a tightly wound 100 turn coil of radius 10 cm, carrying a current of 1 A. What is the magnitude of the magnetic field at the centre of the coil ? **[NCERT]** (Ans. 6.28×10^{-4} T)
2. A circular loop of one turn carries a current of 5.0 A. If the magnetic field at the centre is 0.20 mT, find the radius of the loop. (Ans. 1.57 cm)
3. What current has to be maintained in a circular coil of wire of 50 turns and 2.54 cm in radius in order to just cancel the effect of earth's magnetic field at a place where the horizontal component of earth's field is 1.86×10^{-5} T ? (Ans. 0.015 A)
4. A semicircular arc of radius 20 cm carries a current of 10 A. Calculate the magnitude of the magnetic field at the centre of the arc. **[CBSE D 02]** (Ans. 1.57×10^{-5}T)
5. An alpha particle moves along a circular path of radius 1.0×10^{-10} m with a uniform speed of $2\times10^{6}\,\text{ms}^{-1}$. Calculate the magnetic field produced at the centre of orbit. (Ans. 13.4 T)
6. The electron in hydrogen atom moves around the proton with a speed of $2.2\times10^{6}\,\text{ms}^{-1}$ in a circular orbit of 5.3×10^{-11} m. Calculate (*i*) the equivalent current (*ii*) equivalent dipole moment and (*iii*) the magnetic field at the site of the proton. [Ans. (*i*) 1.057×10^{-3} A (*ii*) 9.32×10^{-24} Am^{-2} (*iii*) 12.5 T]
7. A circular coil has 35 turns and a mean radius of 4.0 cm. It carries a current of 1.2 A. Find the magnetic field (*i*) at a point on the axis of the coil at a distance of 40 cm from its centre and (*ii*) at the centre of the coil. [Ans. (*i*) 6.5×10^{-7} T (*ii*) 6.6×10^{-4} T]
8. A thick straight copper wire, carrying a current of 10 A is bent into a semicircular arc of radius 7.0 cm as shown in Fig. 4.37(*a*). (*i*) State the direction and calculate the magnitude of magnetic field at the centre of arc. (*ii*) How would your answer change if the same wire were bent into a semicircular arc of the same radius but in opposite way as shown in Fig. 4.37(*b*) ? **[CBSE Sample Paper 98]** [Ans. (*i*) 4.5×10^{-5}T, outside the plane of paper, (*ii*) 4.5×10^{-5}T, into the plane of paper]



(*a*) (*b*)

Fig. 4.37

9. A long wire is bent as shown in Fig. 4.38. Find the magnitude and direction of the magnetic field at the centre O of the circular part, if a current I is passed through the wire. [Ans. $\frac{\mu_0}{2R}\left(1-\frac{1}{\pi}\right)$ normally into the plane of paper]

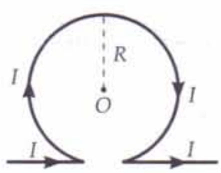


Fig. 4.38

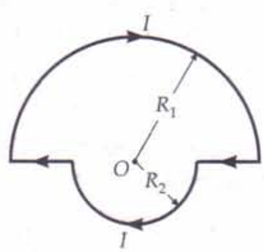


Fig. 4.39

10. Figure 4.39 shows two semicircular loops of radii R_1 and R_2 carrying current I. Find the magnitude and direction of the magnetic field at the common centre O. **[Ans.** $\frac{\mu_0 I}{4}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$, normally downward]

11. A circular segment of radius 10 cm subtends an angle of 60° at its centre. A current of 9 A is flowing through it. Find the magnitude and direction of the magnetic field produced at the centre (Fig. 4.40).

(**Ans.** 9.42×10^{-6} T)

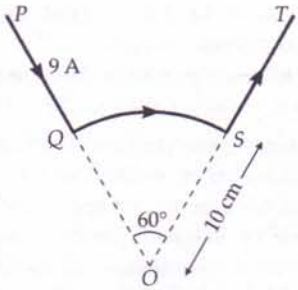


Fig. 4.40

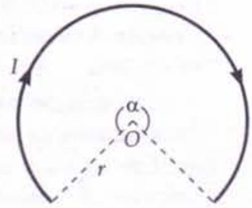


Fig. 4.41

12. A current of I ampere is flowing through the bent wire shown in Fig. 4.41. Find the magnitude and direction of the magnetic field at point O.

$\left(\text{Ans. } B = \frac{\mu_0}{4\pi}\frac{I\alpha}{r}, \text{directed normally downward}\right)$

13. In Fig. 4.42, the curved portion is a semi-circle and the straight wires are long. Find the magnetic field at the point O.

$\left[\text{Ans. } \frac{\mu_0 I}{2d}\left(1+\frac{2}{\pi}\right)\right]$

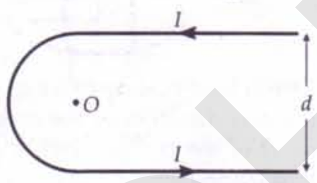


Fig. 4.42

14. Two identical coils each of radius R and having number of turns N are lying in perpendicular planes, such that they have common centre. Find the magnetic field at the centre of the coils, if they carry currents equal to I and $\sqrt{3}I$ respectively.

(Ans. $\mu_0 NI/R$)

15. A metallic wire is bent into the shape shown in Fig. 4.43 and carries a current I. If O is the common centre of all the three circular arcs of radii r, $2r$ and $3r$, find the magnetic field at the point O.

$\left(\text{Ans. } \frac{5\mu_0 I}{24\pi r}\theta, \text{ normally inward}\right)$

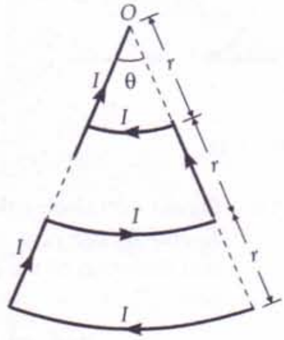


Fig. 4.43

HINTS

1. As the coil is tightly wound, so radius of each turn, $r = 10 \text{ cm} = 0.1 \text{ m}$

$$B = \frac{\mu_0 NI}{2r} = \frac{4\pi \times 10^{-7} \times 100 \times 1}{2 \times 0.1}$$
$$= 2\pi \times 10^{-4} = \mathbf{6.28 \times 10^{-4}\ T}.$$

2. Radius, $r = \frac{\mu_0 NI}{2B} = \frac{4\pi \times 10^{-7} \times 1 \times 5.0}{2 \times 0.20 \times 10^{-3}}$
$$= 1.57 \times 10^{-2} \text{ m} = \mathbf{1.57\ cm}.$$

3. $B = B_H$ or $\frac{\mu_0 NI}{2r} = B_H$

or $\frac{4\pi \times 10^{-7} \times 50 \times I}{2 \times 2.54 \times 10^{-2}} = 1.86 \times 10^{-5}$

$$I = \frac{1.86 \times 2 \times 2.54}{4\pi \times 50} = \mathbf{0.015\ A.}$$

4. Use $B = \frac{\mu_0 I}{4r}$.

5. Take charge on α-particle = + 2e and proceed as in Example 12 on page 4.14.

6. (*i*) Period of revolution, $T = \frac{2\pi r}{v}$

Equivalent current, $I = \frac{e}{T} = \frac{ev}{2\pi r}$

$$= \frac{1.6 \times 10^{-19} \times 2.2 \times 10^6}{2 \times 3.14 \times 5.3 \times 10^{-11}} = \mathbf{1.057 \times 10^{-3}\ A}$$

(*ii*) Equivalent dipole moment,

$$m = IA = I \times \pi r^2$$
$$= 1.057 \times 10^{-3} \times 3.14 \times (5.3 \times 10^{-11})^2$$
$$= \mathbf{9.32 \times 10^{-24}\ Am^2}.$$

(*iii*) $B = \frac{\mu_0 I}{2r} = \frac{4\pi \times 10^{-7} \times 1.057 \times 10^{-3}}{2 \times 5.3 \times 10^{-11}} = \mathbf{12.5\ T.}$

7. (*i*) $N = 35,\ I = 1.2 \text{ A},\ a = 4.0 \text{ cm} = 0.04 \text{ m},$
$r = 40 \text{ cm} = 0.40 \text{ m}$

$$B_{axial} = \frac{\mu_0 N I a^2}{2(a^2 + r^2)^{3/2}} = \frac{4\pi \times 10^{-7} \times 35 \times 1.2 \times (0.04)^2}{2[(0.04)^2 + (0.40)^2]^{3/2}}$$
$$= \frac{4 \times 3.14 \times 10^{-7} \times 35 \times 1.2 \times 0.0016}{2 \times 0.1616 \times 0.402}$$
$$= \mathbf{6.5 \times 10^{-7}\ T.}$$

(*ii*) $B_{centre} = \frac{\mu_0 NI}{2a} = \mathbf{6.6 \times 10^{-4}\ T.}$

8. (*i*) Magnetic field at the centre of the arc is

$$B = \frac{\mu_0 I}{4r}$$

Here $I = 10$ A, $r = 7 \text{ cm} = 0.07$ m,
$\mu_0 = 4\pi \times 10^{-7} \text{ TmA}^{-1}$

$\therefore \quad B = \frac{4\pi \times 10^{-7} \times 10}{4 \times 0.07} = \mathbf{4.5 \times 10^{-5}\ T}.$

The direction of the field is normally outside the plane of paper.

(*ii*) $B = 4.5 \times 10^{-4}$ **T**. The field B will point normally into the plane of paper.

9. Magnitude of the magnetic field at O due to the straight part of the wire is

$$B_1 = \frac{\mu_0}{2\pi} \cdot \frac{I}{R}, \text{ normally out of the plane of paper}$$

Magnetic field at the centre O due to the current loop of radius R is

$$B_2 = \frac{\mu_0 I}{2R}, \text{ normally into the plane of paper}$$

Resultant field at O is

$$B = B_2 - B_1 = \frac{\mu_0 I}{2R}\left(1 - \frac{1}{\pi}\right),$$

normally into the plane of paper.

10. $B = B_1 + B_2 = \frac{\mu_0 I}{4R_1} + \frac{\mu_0 I}{4R_2} = \frac{\mu_0 I}{4}\left(\frac{1}{R_1} + \frac{1}{R_2}\right).$

11. Here $\theta = 60° = \frac{\pi}{3}$ rad

As $\theta\,(\text{rad}) = \frac{l}{r} \quad \therefore \quad \frac{\pi}{3} = \frac{l}{r} \quad \text{or} \quad l = \frac{\pi r}{3}$

According to Biot-Savart law, magnetic field at the centre O is

$$B = \frac{\mu_0}{4\pi} \cdot \frac{Il}{r^2} = \frac{\mu_0}{4\pi} \cdot \frac{I}{r^2} \cdot \frac{\pi r}{3} = \frac{\mu_0}{4\pi} \cdot \frac{\pi}{3} \cdot \frac{I}{r}$$

$$= \frac{10^{-7} \times 3.14 \times 9}{3 \times 0.10} = \mathbf{9.42 \times 10^{-6}\ T.}$$

12. Any element $\vec{dl}$ on the arc will be perpendicular to the position vector $\vec{r}$, so the field due to one such element at the centre O will be

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin \pi/2}{r^2} = \frac{\mu_0}{4\pi} \cdot \frac{Idl}{r^2}$$

Magnetic field due to the entire arc at the centre O,

$$B = \int dB = \frac{\mu_0 I}{4\pi r^2} \int dl = \frac{\mu_0 I}{4\pi r^2} \cdot l$$

But l = length of arc = αr

$$\therefore \quad B = \frac{\mu_0 I}{4\pi r^2} \cdot \alpha r = \frac{\mu_0 I \alpha}{4\pi r}.$$

13. Magnetic field at point O due to any current element is perpendicular to and points out of the plane of paper.

Magnetic field at O due to the upper straight wire is

$$B_1 = \frac{1}{2} \times \frac{\mu_0 I}{2\pi (d/2)} = \frac{\mu_0 I}{2\pi d}$$

Similarly, field at O due to lower straight wire is

$$B_2 = \frac{\mu_0 I}{2\pi d}$$

Field at O due to the semicircle of radius $d/2$ is

$$B_3 = \frac{1}{2} \times \frac{\mu_0 I}{2(d/2)} = \frac{\mu_0 I}{2d}$$

Resultant field at O,

$$B = B_1 + B_2 + B_3 = \frac{\mu_0 I}{2d}\left[1 + \frac{2}{\pi}\right].$$

14. Magnetic fields produced by the two coils at their common centre are

$$B_1 = \frac{\mu_0 NI}{2R} \quad \text{and} \quad B_2 = \frac{\mu_0 N\sqrt{3}I}{2R}$$

The planes of the two coils are perpendicular to each other. So the fields B_1 and B_2 will also be perpendicular to each other, as shown in Fig. 4.44.

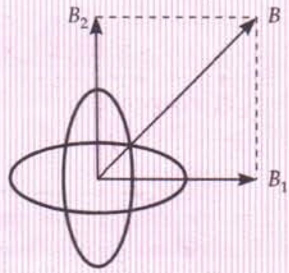


Fig. 4.44

The resultant field at the common centre is

$$B = \sqrt{B_1^2 + B_2^2}$$

$$= \left[\left(\frac{\mu_0 NI}{2R}\right)^2 + \left(\frac{\sqrt{3}\,\mu_0 NI}{2R}\right)^2\right]^{1/2}$$

$$= \frac{\mu_0 NI}{2R}(1+3)^{1/2} = \frac{\mu_0 NI}{R}.$$

15. Magnetic field at O due to the straight parts of the wire will be zero. Magnetic fields at O due to the three circular arcs of radii r, $2r$ and $3r$ are

$$B_1 = \frac{\mu_0 I}{4\pi} \cdot \frac{\theta}{r}, \quad \text{acting normally inward}$$

$$B_2 = \frac{\mu_0 I}{4\pi} \cdot \frac{\theta}{2r}, \text{ acting normally outward}$$

$$B_3 = \frac{\mu_0 I}{4\pi} \cdot \frac{\theta}{3r}, \text{ acting normally inward}$$

Thus the total magnetic field at the centre O is

$$B = B_1 - B_2 + B_3 = \frac{\mu_0 I}{4\pi}\left(\frac{\theta}{r} - \frac{\theta}{2r} + \frac{\theta}{3r}\right)$$

$$= \frac{5\mu_0 I}{24\pi r}\theta, \text{ acting normally inward.}$$

4.8 AMPERE'S CIRCUITAL LAW AND ITS APPLICATION TO INFINITELY LONG STRAIGHT WIRE

9. *(a) State Ampere's circuital law and prove it for the magnetic field produced by a straight current carrying conductor.*

Ampere's circuital law. Just as Gauss's law is an alternative form of Coulomb's law in electrostatics, similarly we have Ampere's circuital law as an alternative form of Biot-Savart law in magnetostatics. Ampere's circuital law gives a relationship between the line integral of a magnetic field B and the total current I which produces this field.

Ampere's circuital law states that the line integral of the magnetic field $\vec{B}$ around any closed circuit is equal to μ_0 (permeability constant) times the total current I threading or passing through this closed circuit. Mathematically,

$$\oint \vec{B}.\vec{dl} = \mu_0 I$$

In a simplified form, Ampere's circuital law states that if field $\vec{B}$ is directed along the tangent to every point on the perimeter L of a closed curve and its magnitude is constant along the curve, then

$$BL = \mu_0 I$$

where I is the net current enclosed by the closed circuit. The closed curve is called ***Amperean loop*** which is a *geometrical entity* and *not a* real wire loop.

Proof for a straight current carrying conductor. Consider an infinitely long straight conductor carrying a current I. From Biot-Savart law, the magnitude of the magnetic field $\vec{B}$ due to the current carrying conductor at a point, distant r from it is given by

$$B = \frac{\mu_0 I}{2\pi r}$$

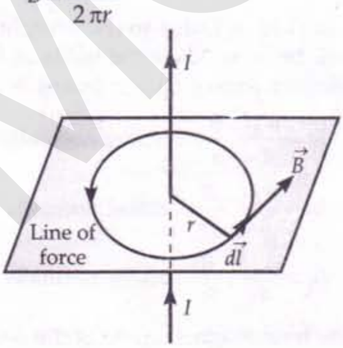


Fig. 4.45 Ampere's circuital law.

As shown in Fig. 4.45, the field $\vec{B}$ is directed along the circumference of the circle of radius r with the wire as centre. The magnitude of the field $\vec{B}$ is same for all points on the circle. To evaluate the line integral of the magnetic field $\vec{B}$ along the circle, we consider a small current element $\vec{dl}$ along the circle. At every point on the circle, both $\vec{B}$ and $\vec{dl}$ are tangential to the circle so that the angle between them is zero.

$$\therefore \quad \vec{B}.\vec{dl} = B\,dl \cos 0° = B\,dl$$

Hence the line integral of the magnetic field along the circular path is

$$\oint \vec{B}.\vec{dl} = \oint B\,dl = B\oint dl = \frac{\mu_0 I}{2\pi r}.l$$

$$= \frac{\mu_0 I}{2\pi r}.2\pi r$$

$$\therefore \oint \vec{B}.\vec{dl} = \mu_0 I$$

This proves Ampere's law. This law is valid for any assembly of current and for any arbitrary closed loop.

9. *(b) Calculate, using Ampere's circuital theorem, the magnetic field due to an infinitely long wire carrying a current I.*

Application of Ampere's law to a straight conductor. Fig. 4.46 shows a circular loop of radius r around an infinitely long straight wire carrying current I. As the field lines are circular, the field $\vec{B}$ at any point of the circular loop is directed along the tangent to the circle at that point. By symmetry, the magnitude of field $\vec{B}$ is same at every point of the circular loop. Therefore,

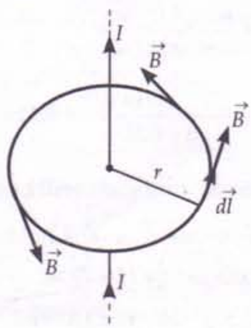


Fig. 4.46

$$\oint \vec{B}.\vec{dl} = \oint B\,dl \cos 0° = B\oint dl = B.2\pi r$$

From Ampere's circuital law,

$$B.2\pi r = \mu_0 I$$

$$\therefore \quad B = \frac{\mu_0 I}{2\pi r}$$

For Your Knowledge

- Ampere's circuital law is not independent of the Biot-Savart law. It can be derived from the Biot-Savart law. Its relationship to the Biot-Savart law is similar to the relationship between Gauss's law and Coulomb's law.
- Both Ampere's circuital law and Biot-Savart law relate magnetic field to the electric current.
- Ampere's and Gauss's laws relate one physical quantity (magnetic or electric quantity) on the boundary or periphery to another physical quantity (current or charge), called source, in the interior.
- Ampere's circuital law holds for steady currents which do not change with time.
- Although both Ampere's law and Biot-Savart law are equivalent in physical content, yet the Ampere's law is more useful under certain symmetrical situations. The mathematics of finding the magnetic field of a solenoid and toroid becomes much simpler if we apply Ampere's law.

4.9 MAGNETIC FIELD INSIDE A STRAIGHT SOLENOID

10. *Give a qualitative discussion of the magnetic field produced by a straight solenoid. Apply Ampere's circuital law to calculate magnetic field inside a straight solenoid.*

Magnetic field of a straight solenoid : A qualitative discussion. *A solenoid means an insulated copper wire wound closely in the form of a helix*. The word solenoid comes from a Greek word meaning *channel* and was first used by Ampere. By a long solenoid, we mean that the length of the solenoid is very large as compared to its diameter.

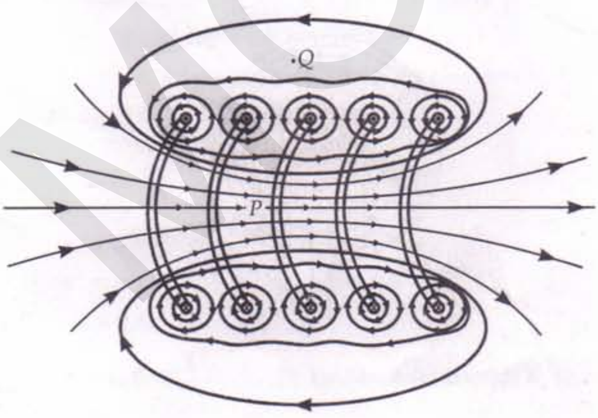


Fig 4.47 Magnetic field due to a section of a finite solenoid.

Figure 4.47 shows an enlarged view of the magnetic field due to a section of a solenoid. At various turns of the solenoid, current enters the plane of paper at points marked ⊗ and leaves the plane of paper at points marked ⊙. The magnetic field at points close to a single turn of the solenoid is in the form of concentric circles like that of a straight current carrying wire. The resultant field of the solenoid is the vector sum of the fields due to all the turns of the solenoid. Obviously the fields due to the neighbouring turns add up along the axis of the solenoid but they cancel out in the perpendicular direction. At outside points such as Q, the fields of the points marked ⊗ tend to cancel out the fields of the points marked ⊙. Thus the field at interior midpoint P is uniform and strong. The field at the exterior midpoint Q is weak and is along the axis of the solenoid with no perpendicular component. Fig. 4.48 shows the field pattern of a solenoid of finite length.

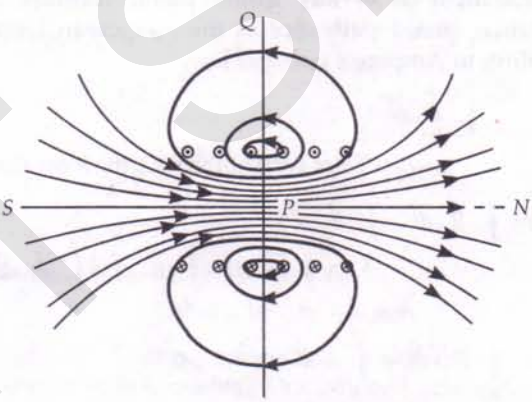


Fig 4.48 Magnetic field of a finite solenoid.

The polarity of any end of the solenoid can be determined by using *clock rule or Ampere's right hand rule*.

Ampere's right hand rule. *Grasp the solenoid with the right hand so that the fingers point along the direction of the current, the extended thumb will then indicate the face of the solenoid that has north polarity* (Fig. 4.49).

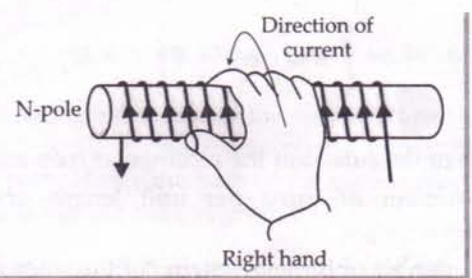


Fig. 4.49 Ampere's rule for polarity of a solenoid.

Calculation of magnetic field inside a long straight solenoid. The magnetic field inside a closely wound long solenoid is uniform everywhere and zero outside

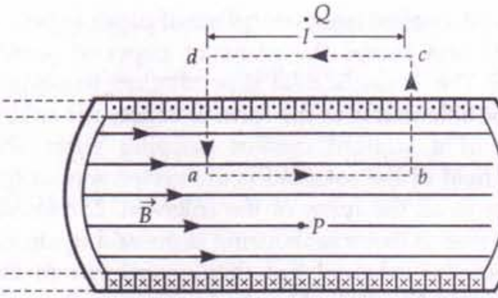


Fig. 4.50 The magnetic field of a very long solenoid.

it. Fig. 4.50 shows the sectional view of a long solenoid. At various turns of the solenoid, current comes out of the plane of paper at points marked ⊙ and enters the plane of paper at points marked ⊗. To determine the magnetic field $\vec{B}$ at any inside point, consider a rectangular closed path *abcd* as the Amperean loop. According to Ampere's circuital law,

$$\oint \vec{B}\cdot\vec{dl}$$
$$= \mu_0 \times \text{Total current through the loop } abcd$$

Now $$\oint \vec{B}\cdot\vec{dl} = \int_a^b \vec{B}\cdot\vec{dl} + \int_b^c \vec{B}\cdot\vec{dl} + \int_c^d \vec{B}\cdot\vec{dl} + \int_d^a \vec{B}\cdot\vec{dl}$$

But $$\int_b^c \vec{B}\cdot\vec{dl} = \int_b^c B\,dl\cos 90^\circ = 0$$

$$\int_d^a \vec{B}\cdot\vec{dl} = \int_d^a B\,dl\cos 90^\circ = 0$$

$$\int_c^d \vec{B}\cdot\vec{dl} = 0$$

as $B = 0$ for points outside the solenoid.

$$\therefore \quad \oint \vec{B}\cdot\vec{dl} = \int_a^b \vec{B}\cdot\vec{dl}$$

$$= \int_a^b B\,dl\cos 0^\circ = B\int_a^b dl = Bl$$

where,

l = length of the side *ab* of the rectangular loop *abcd*.

Let number of turns per unit length of the solenoid $= n$

Then number of turns in length l of the solenoid

$$= nl$$

Thus the current I of the solenoid threads the loop *abcd*, *nl* times.

$\therefore$ Total current threading the loop $abcd = nlI$

Hence $Bl = \mu_0 nlI$ or $B = \mu_0 nI$

It can be easily shown that the magnetic field at the end of the solenoid is just one half of that at its middle. Thus

$$B_{\text{end}} = \frac{1}{2}\mu_0 nI$$

Figure 4.51 shows the variation of magnetic field on the axis of a long straight solenoid with distance x from its centre.

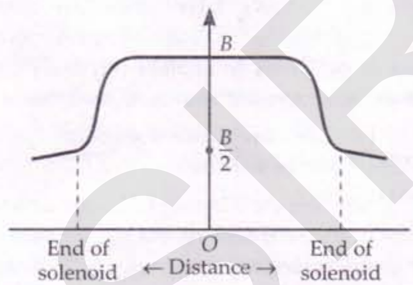


Fig. 4.51 Variation of magnetic field along the axis of solenoid.

4.10 MAGNETIC FIELD DUE TO A TOROIDAL SOLENOID

11. *Apply Ampere's circuital law to find the magnetic field both inside and outside of a toroidal solenoid.*

Magnetic field due to a toroidal solenoid. *A solenoid bent into the form of a closed ring is called a toroidal solenoid*. Alternatively, it is an anchor ring (torous) around which a large number of turns of a metallic wire are wound, as shown in Fig. 4.52. We shall see that the magnetic field $\vec{B}$ has a constant magnitude everywhere inside the toroid while it is zero in the open space interior (point P) and exterior (point Q) to the toroid.

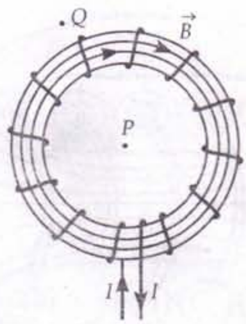


Fig. 4.52 A toroidal solenoid.

Figure 4.53 shows a sectional view of the toroidal solenoid. The direction of the magnetic field inside is clockwise as per the right-hand thumb rule for circular loops. Three circular Amperean loops are shown by

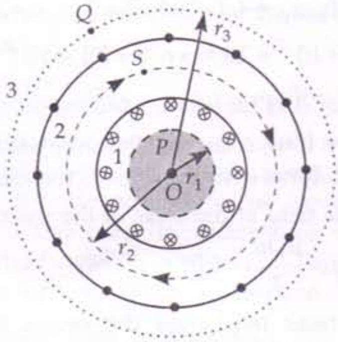


Fig. 4.53 A sectional view of the toroidal solenoid.

dashed lines. By symmetry, the magnetic field should be tangential to them and constant in magnitude for each of the loops.

1. *For points in the open space interior to the toroid.* Let B_1 be the magnitude of the magnetic field along the Amperean loop 1 of radius r_1.

Length of the loop 1, $L_1 = 2\pi r_1$

As the loop encloses no current, so $I = 0$

Applying Ampere's circuital law,

$$B_1 L_1 = \mu_0 I$$

or $$B_1 \times 2\pi r_1 = \mu_0 \times 0$$

or $$B_1 = 0$$

Thus the magnetic field at any point P in the open space interior to the toroid is zero.

2. *For points inside the toroid.* Let B be the magnitude of the magnetic field along the Amperean loop 2 of radius r.

Length of loop 2, $L_2 = 2\pi r$

If N is the total number of turns in the toroid and I the current in the toroid, then total current enclosed by the loop 2 = NI

Applying Ampere's circuital law,

$$B \times 2\pi r = \mu_0 \times NI$$

or $$B = \frac{\mu_0 NI}{2\pi r}$$

If r be the average radius of the toroid and n the number of turns per unit length, then

$$N = 2\pi r n$$

$$\therefore \quad B = \mu_0 nI$$

3. *For points in the open space exterior to the toroid.* Each turn of the toroid passes twice through the area enclosed by the Amperean loop 3. But for each turn, the current coming out of the plane of paper is cancelled by the current going into the plane of paper. Thus, $I = 0$ and hence $B_3 = 0$.

For Your Knowledge

- The magnetic field inside a toroidal solenoid is independent of its radius and depends only on the current and the number of turns per unit length. The field inside the toroid has constant magnitude and tangential direction at every point.
- In ideal toroid, the coils are circular and magnetic field is zero external to the toroid. In a real toroid, the turns form a helix and there is a small magnetic field external to the toroid.
- Toroids are expected to play a key role in the ***Tokamak*** which acts as a magnetic container for the fusion of plasma in fusion (thermonuclear) power reactors.

Examples based on

Ampere's Circuital Law and Magnetic Field due to (*i*) Straight Solenoid (*ii*) Toroidal Solenoid

Formulae Used

1. Ampere's circuital law, $\oint \vec{B}\, \vec{dl} = \mu_0 I$

   When B is directed along tangent to every point on closed curve L, $BL = \mu_0 I$
2. Magnetic field due to straight solenoid,

   (*i*) At a point well inside the solenoid, $B = \mu_0 nI$

   (*ii*) At either end of the solenoid, $B_{end} = \frac{1}{2}\mu_0 nI$

   Here n is the number of turns per unit length.
3. Magnetic field inside a toroidal solenoid, $B = \mu_0 nI$

   Magnetic field is zero outside the toroid.

Units Used

B is in tesla, current I in ampere and n in m^{-1}.

Example 27. *A solenoid coil of 300 turns/m is carrying a current of 5 A. The length of the solenoid is 0.5 m and has a radius of 1 cm. Find the magnitude of the magnetic field inside the solenoid.* [CBSE F 04]

Solution. Here $n = 300$ turns/m, $I = 5$ A

$$\therefore \quad B = \mu_0 nI = 4\pi \times 10^{-7} \times 300 \times 5 = \mathbf{1.9 \times 10^{-3}\ T}.$$

Example 28. *A solenoid of length 0.5 m has a radius of 1 cm and is made up of 500 turns. It carries a current of 5 A. What is the magnitude of the magnetic field inside the solenoid ?* [NCERT]

Solution. Number of turns per unit length,

$$n = \frac{N}{l} = \frac{500}{0.5\ \text{m}} = 1000\ \text{turns / m}$$

Here $l = 0.5$ m and $r = 0.01$ m *i.e.*, $l >> a$. So we can use formula for magnetic field inside a long solenoid.

$$B = \mu_0 nI = 4\pi \times 10^{-7} \times 1000 \times 5 = \mathbf{6.28 \times 10^{-3}\ T}.$$

Example 29. *A 0.5 m long solenoid has 500 turns and has a flux density of 2.52×10^{-3} T at its centre. Find the current in the solenoid. Given $\mu_0 = 4\pi \times 10^{-7}$ Hm^{-1}.* **[ISCE 95]**

Solution. Number of turns per unit length,

$$n = \frac{N}{l} = \frac{500}{0.5 \text{ m}} = 1000 \text{ turns / m}$$

As $\quad B = \mu_0 n I$

$$\therefore \quad I = \frac{B}{\mu_0 n} = \frac{2.52 \times 10^{-3}}{4\pi \times 10^{-7} \times 1000} = \mathbf{2.0\ A.}$$

Example 30. *A copper wire having a resistance of $0.01\,\Omega$ per metre is used to wind a 400 turn solenoid of radius 1.0 cm and length 20 cm. Find the emf of a battery which when connected across the solenoid would produce a magnetic field of 10^{-2} T near the centre of the solenoid.*

Solution. Length of wire used

$$= 2\pi r \times \text{No. of turns}$$
$$= 2\pi \times 1.0 \times 10^{-2} \times 400 \text{ m}$$

Resistance per unit length $= 0.01\,\Omega\text{ m}^{-1}$

$\therefore$ Total resistance of wire,

$$R = 2\pi \times 1.0 \times 10^{-2} \times 400 \times 0.01$$
$$= 8\pi \times 10^{-2}\,\Omega$$

No. of turns per unit length,

$$n = \frac{400}{20 \times 10^{-2}} = 2000 \text{ m}^{-1}$$

As $\quad B = \mu_0 n I = \mu_0 n \dfrac{\varepsilon}{R}$

$$\therefore \quad \varepsilon = \frac{BR}{\mu_0 n} = \frac{10^{-2} \times 8\pi \times 10^{-2}}{4\pi \times 10^{-7} \times 2000} = \mathbf{1\ V.}$$

Example 31. *A solenoid 50 cm long has 4 layers of windings of 350 turns each. The radius of the lowest layer is 1.4 cm. If the current carried is 6.0 A, estimate the magnitude of $\vec{B}$ (a) near the centre of the solenoid on its axis and off its axis, (b) near its ends on its axis, (c) outside the solenoid near its centre.*

Solution. (*a*) The magnitude of the magnetic field at or near the centre of the solenoid is given by

$$B = \mu_0 n I$$

where n is the number of turns per unit length. This expression for $\vec{B}$ can also be used if the solenoid has more than one layer of windings because the radius of the wire does not enter this equation. Therefore,

$$n = \frac{\text{No. of turns per layer} \times \text{No. of layers}}{\text{Length of the solenoid}}$$

$$= \frac{350 \times 4}{0.50} = 2800 \text{ m}^{-1}$$

Now $I = 6.0$ A, $\mu_0 = 4\pi \times 10^{-7}$ TmA^{-1}, $n = 2800$ m^{-1}

$$\therefore \quad B = 4\pi \times 10^{-7} \times 2800 \times 6 \text{ T} = \mathbf{2.1 \times 10^{-2}\ T}$$

This value of $\vec{B}$ is for *both* on and off the axis, since for an infinitely long solenoid, the internal field near the centre is uniform over the entire cross-section.

(*b*) Magnetic field at the ends of the solenoid is

$$B_{end} = \frac{\mu_0 n I}{2} = \mathbf{1.05 \times 10^{-2}\ T.}$$

(*c*) The outside field near the centre of a long solenoid is negligible compared to the internal field.

Example 32. *A coil wrapped around a toroid has inner radius of 20.0 cm and an outer radius of 25.0 cm. If the wire wrapping makes 800 turns and carries a current of 12.0 A, what are the maximum and minimum values of the magnetic field within the toroid ?*

Solution. Let a and b denote the inner and outer radii of the toroid. Then

$$B_{max} = \mu_0 n I = \mu_0 \frac{N}{2\pi a} I = \frac{4\pi \times 10^{-7} \times 800 \times 12.0}{2\pi \times 20.0 \times 10^{-2}}$$
$$= 9.6 \times 10^{-3} \text{ T} = \mathbf{9.6\ mT.}$$

$$B_{min} = \mu_0 n I = \mu_0 \frac{N}{2\pi b} I = \frac{4\pi \times 10^{-7} \times 800 \times 12.0}{2\pi \times 25.0 \times 10^{-2}}$$
$$= 7.68 \times 10^{-3} \text{ T} = \mathbf{7.68\ mT.}$$

Example 33. *(i) A straight thick long wire of uniform cross-section of radius 'a' is carrying a steady current I. Use Ampere's circuital law to obtain a relation showing the variation of the magnetic field (B_r) inside and outside the wire with distance r, ($r \le a$) and ($r > a$) of the field point from the centre of its cross-section. Plot a graph showing the variation of field B with distance r.*

(ii) Calculate the ratio of magnetic field at a point $a/2$ above the surface of the wire to that at a point $a/2$ below its surface. What is the maximum value of the field of this wire ?

[NCERT ; CBSE D 10]

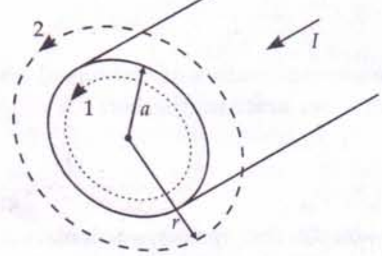


Fig. 4.54 A steady current I distributed uniformly across a wire of radius a.

Solution. (*i*) Application of Ampere's law to a long straight cylindrical wire. By symmetry, the magnetic lines of force will be circles, with their centres on the axis of the cylinder and in planes perpendicular to the axis of the cylinder. So we consider Amperean loop as a circle of radius r.

Field at outside points. The Amperean loop is a circle labelled 2 having radius $r > a$.

Length of the loop, $L = 2\pi r$

Net current enclosed by the loop = I

By Ampere's circuital law,

$$BL = \mu_0 I$$

or $$B \times 2\pi r = \mu_0 I$$

or $$B = \frac{\mu_0 I}{2\pi r} \qquad \text{[For } r > a\text{]}$$

i.e., $$B \propto \frac{1}{r} \qquad \text{[For outside points]}$$

Field at inside points. The Amperean loop is a circle labelled 1 with $r < a$.

Length of the loop, $L = 2\pi r$

Clearly, the current enclosed by loop 1 is less than I. As the current distribution is uniform, the fraction of I enclosed is

$$I' = \frac{I}{\pi a^2} \times \pi r^2 = \frac{Ir^2}{a^2}$$

Applying Ampere's law,

$$BL = \mu_0 I'$$

or $$B \times 2\pi r = \mu_0 \frac{Ir^2}{a^2}$$

or $$B = \left(\frac{\mu_0 I}{2\pi a^2}\right) r \qquad \text{[For } r < a\text{]}$$

i.e., $$B \propto r \qquad \text{[For inside points]}$$

Thus the field B is proportional to r as we move from the axis of the cylinder towards its surface and then it decreases as $\frac{1}{r}$. The variation of B with distance r from the centre of the wire is shown in Fig. 4.55(a).

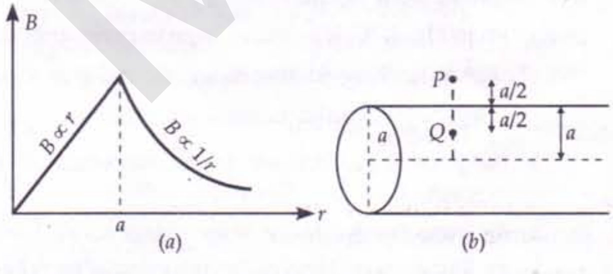


Fig. 4.55 (a) Sketch of the magnitude of the magnetic field for the long conductor of radius a.

(ii) Suppose the point P lies at distance $a/2$ above the surface of the wire and point Q lies at distance $a/2$ below the surface. [Fig. 4.55(b)]

Magnetic field at point P at distance $r = 3a/2$ from the axis of the wire is

$$B_P = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 I}{2\pi(3/2)\, a} = \frac{\mu_0 I}{3\pi a}$$

Magnetic field at point Q at distance $r = a/2$ from the axis of the wire is

$$B_Q = \frac{\mu_0 Ir}{2\pi a^2} = \frac{\mu_0 I}{2\pi a^2}\left(\frac{a}{2}\right) = \frac{\mu_0 I}{4\pi a}$$

$$\therefore \quad \frac{B_P}{B_Q} = \frac{\mu_0 I}{3\pi a} \times \frac{4\pi a}{\mu_0 I} = \mathbf{4:3.}$$

Clearly, B is maximum on the surface of the wire *i.e.,* at $r = a$. Hence,

$$B_{max} = \frac{\mu_0 I}{2\pi a}$$

Example 34. *A wire of radius 0.5 cm carries a current of 100 A, which is uniformly distributed over its cross-section. Find the magnetic field (i) at 0.1 cm from the axis of the wire, (ii) at the surface of the wire and (iii) at a point outside the wire 0.2 cm from the surface of the wire.*

Solution. Here $R = 0.5$ cm $= 0.5 \times 10^{-2}$ m, $I = 100$ A

We use the results of the above example.

(i) $$B_{inside} = \frac{\mu_0 I}{2\pi R^2} \cdot r = \frac{4\pi \times 10^{-7} \times 100 \times 0.1 \times 10^{-2}}{2\pi \times (0.5 \times 10^{-2})^2} = \mathbf{8.0 \times 10^{-4}\ T.}$$

(ii) $$B_{surface} = \frac{\mu_0 I}{2\pi R} = \frac{4\pi \times 10^{-7} \times 100}{2\pi \times 0.5 \times 10^{-2}} = \mathbf{4.0 \times 10^{-3}\ T.}$$

(iii) Here $r = 0.5 + 0.2 = 0.7$ cm $= 0.7 \times 10^{-2}$ m

$$B_{outside} = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 100}{2\pi \times 0.7 \times 10^{-2}} = \mathbf{2.86 \times 10^{-5} T.}$$

Problems For Practice

1. A long solenoid consists of 20 turns per cm. What current is necessary to produce a magnetic field of 20 mT inside the solenoid? (Ans. 8.0 A)
2. A long solenoid is made by closely winding a wire of radius 0.5 mm over a cylindrical non-magnetic frame so that successive turns nearly touch each other. What will be the magnetic field at the centre of the solenoid if a current of 5 A flows through it? (Ans. $2\pi \times 10^{-3}$ T)

3. The magnetic field at the centre of a 50 cm long solenoid is 4.0×10^{-2} T when a current of 8.0 A flows through it. What is the number of turns in the solenoid ? Take $\pi = 3.14$. (Ans. 1990)

4. A solenoid is 1.0 m long and 3.0 cm in diameter. It has five layers of windings of 850 turns each and carries a current of 5.0 A. (*i*) What is B at its centre ? (*ii*) What is the magnetic flux ϕ_B for a cross-section of the solenoid at the centre ?

[Ans. (*i*) 2.67×10^{-2} T, (*ii*) 1.9×10^{-5} Wb]

5. A solenoid is 2.0 m long and 3.0 cm in diameter. It has 5 layers of winding of 1000 turns each and carries a current of 5.0 A. What is the magnetic field at the centre ? Use the standard value of μ_0. **[Punjab 97C]**

(Ans. 1.57×10^{-2} T)

6. A toroid has a core of inner radius 20 cm and outer radius 22 cm around which 4200 turns of a wire are wound. If the current in the wire is 10 A, what is the magnetic field (*i*) inside the core of toroid (*ii*) outside the toroid and (*iii*) in the empty space surrounded by the toroid. [Ans. (*i*) 0.04 T (*ii*) Zero (*iii*) Zero]

7. A long straight solid conductor of radius 4 cm carries a current of 2 A, which is uniformly distributed over its circular cross-section. Find the magnetic field at a distance of 3 cm from the axis of the conductor. (Ans. 7.5×10^{-6} T)

HINTS

1. Here $n = 20\ \text{cm}^{-1} = 20 \times 10^2\ \text{m}^{-1}$,

$B = 20\ \text{mT} = 20 \times 10^{-3}$ T

$$\text{Current, } I = \frac{B}{\mu_0 n} = \frac{20 \times 10^{-3}}{4\pi \times 10^{-7} \times 20 \times 10^2} = \mathbf{8.0\ A.}$$

2. Diameter of the wire $= 2 \times 0.5 = 1.0$ mm $= 10^{-3}$ m

$\therefore$ Number of turns per unit length,

$$n = \frac{1}{10^{-3}\ \text{m}} = 10^3\ \text{m}^{-1}$$

Also, $I = 5$ A, $\mu_0 = 4\pi \times 10^{-7}$ Tm A^{-1}

$$\therefore \quad B = \mu_0 nI = 4\pi \times 10^{-7} \times 10^3 \times 5 = \mathbf{2\pi \times 10^{-3}\ T.}$$

3. $B = \mu_0 nI = \mu_0 \dfrac{N}{l} I$

$$N = \frac{Bl}{\mu_0 I} = \frac{4.0 \times 10^{-2} \times 0.50}{4 \times 3.14 \times 10^{-7} \times 8} = \mathbf{1990.}$$

4. Number of turns per unit length,

$$n = \frac{5 \times 850}{1.0} = 4250\ \text{m}^{-1}$$

(*i*) $B = \mu_0 nI = 4\pi \times 10^{-7} \times 4250 \times 5.0 = \mathbf{2.67 \times 10^{-2}\ T.}$

(*ii*) $\phi_B = BA = B \times \pi r^2$

$= 2.67 \times 10^{-2} \times 3.14 \times (1.5 \times 10^{-2})^2$

$= \mathbf{1.9 \times 10^{-5}\ Wb.}$

5. Number of turns per unit length,

$$n = \frac{5 \times 1000}{2.0} = 2500\ \text{m}^{-1}$$

$$B = \mu_0 nI = 4\pi \times 10^{-7} \times 2500 \times 5.0 = \mathbf{1.57 \times 10^{-2}\ T.}$$

6. Mean radius of toroid,

$$r = \frac{20 + 22}{2} = 21\ \text{cm} = 0.21\ \text{m}$$

Number of turns per unit length

$$= \frac{4200}{2\pi r} = \frac{4200}{2\pi \times 0.21} = \frac{1000}{\pi}\ \text{m}^{-1}$$

(*i*) Field inside the core of the toroid,

$$B = \mu_0 nI = 4\pi \times 10^{-7} \times \frac{1000}{\pi} \times 10 = \mathbf{0.04\ T.}$$

(*ii*) Magnetic field outside the toroid is zero.

(*iii*) Magnetic field in the empty space surrounded by toroid is zero.

7. Current enclosed by the loop of radius r,

$$I' = \frac{I}{\pi R^2} \times \pi r^2 = \frac{Ir^2}{R^2}$$

Using Ampere's circuital law,

$$BL = \mu_0 I'$$

$$B \times 2\pi r = \mu_0 \frac{Ir^2}{R^2} \quad \text{or} \quad B = \frac{\mu_0 Ir}{2\pi R^2}$$

$$= \frac{4\pi \times 10^{-7} \times 2 \times 3 \times 10^{-2}}{2\pi \times (4 \times 10^{-2})^2} = \mathbf{7.5 \times 10^{-6}\ T.}$$

4.11 FORCE ON A MOVING CHARGE IN A MAGNETIC FIELD

12. *State the factors on which the force acting on a charge moving in a magnetic field depends. Write the expression for this force. When is this force minimum and maximum ? Define magnetic field. Also define the SI unit of magnetic field.*

Magnetic force on a moving charge. *The electric charges moving in a magnetic field experience a force,* while there is no such force on static charges. This fact was first recognized by *Hendrik Antoon Lorentz,* a great Dutch physicist, nearly a century ago.

Suppose a positive charge q moves with velocity $\vec{v}$ in a magnetic field $\vec{B}$ and $\vec{v}$ makes an angle θ with $\vec{B}$, as shown in Fig. 4.56. It is found from experiments that the charge q moving in the magnetic field $\vec{B}$ experiences a force $\vec{F}$ such that

1. *the force is proportional to the magnitude of the magnetic field, i.e.,* $F \propto B$
2. *the force is proportional to the charge q, i.e.,* $F \propto q$
3. *the force is proportional to the component of the velocity v in the perpendicular direction of the field B, i.e.,*

$$F \propto v \sin\theta$$

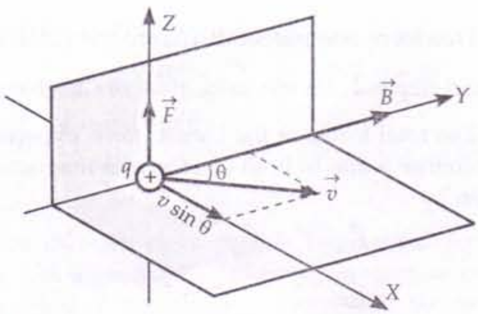


Fig. 4.56 Magnetic Lorentz force.

Combining the above factors, we get

$$F \propto Bqv \sin\theta$$

or $$F = kqvB \sin\theta$$

The unit of magnetic field is so defined that the proportionality constant k becomes unity in the above equation. Thus

$$F = qvB \sin\theta$$

This force deflects the charged particle sideways and is called ***magnetic Lorentz force***. As the direction of $\vec{F}$ is perpendicular to both $\vec{v}$ and $\vec{B}$, so we can express $\vec{F}$ in terms of the vector product of $\vec{v}$ and $\vec{B}$ as

$$\vec{F} = q(\vec{v} \times \vec{B})$$

Figure 4.56 shows the relationship among the directions of vectors $\vec{F}$, $\vec{v}$ and $\vec{B}$. Vectors $\vec{v}$ and $\vec{B}$ lie in the XY-plane. The direction of $\vec{F}$ is perpendicular to this plane and points along + Z-axis *i.e.*, $\vec{F}$ acts in the direction of $\vec{v} \times \vec{B}$.

Special Cases

Case 1. If $v = 0$, then $F = 0$

Thus a stationary charged particle does not experience any force in a magnetic field.

Case 2. If $\theta = 0°$ or $180°$, then $F = 0$

Thus a charged particle moving parallel or antiparallel to a magnetic field does not experience any force in the magnetic field.

Case 3. If $\theta = 90°$, then $F = qvB \sin 90° = qvB$

Thus a charged particle experiences the maximum force when it moves perpendicular to the magnetic field.

Rules for finding the direction of force on a charged particle moving perpendicular to a magnetic field. The direction of magnetic Lorentz force $\vec{F}$ can be determined by using either of the following two rules :

1. ***Fleming's left hand rule.*** *Stretch the thumb and the first two fingers of the left hand mutually perpendicular to each other. If the forefinger points in the direction of the magnetic field, central finger in the direction of current, then the thumb gives the direction of the force on the charged particle.* (Fig. 4.57)

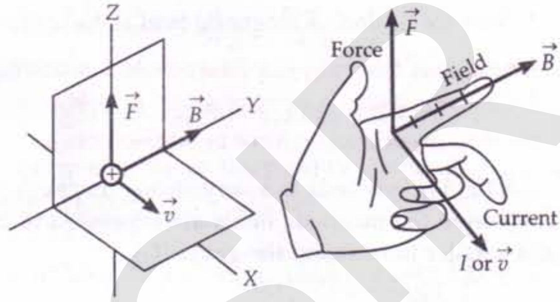


Fig. 4.57 Fleming's left hand rule.

2. ***Right hand (palm) rule.*** *Open the right hand and place it so that tips of the fingers point in the direction of the field* $\vec{B}$ *and thumb in the direction of velocity* $\vec{v}$ *of the positive charge, then the palm faces towards the force* $\vec{F}$, *as shown in Fig. 4.58.*

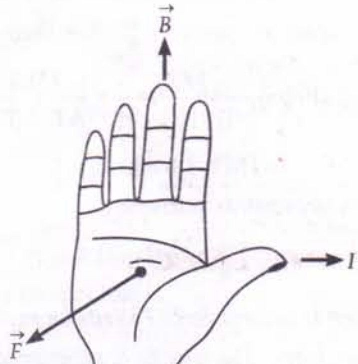


Fig. 4.58 Right hand palm rule.

Definition of magnetic field. We know that

$$B = \frac{F}{qv \sin\theta}$$

If $q = 1$, $v = 1$, $\theta = 90°$, $\sin 90° = 1$, then $B = F$

Thus the magnetic field at a point may be defined as the force acting on a unit charge moving with a unit velocity at right angles to the direction of the field.

SI unit of magnetic field. Again, we use

$$B = \frac{F}{qv \sin\theta}$$

If $F = 1\,N,\ q = 1\,C,\ v = 1\,ms^{-1},\ \theta = 90°$, then

$$\therefore \quad \text{SI unit of } B = \frac{1\,N}{1\,C.1\,ms^{-1}.\sin 90°}$$

$$= \frac{1\,N}{1\,A.1\,m} \qquad [\because Cs^{-1} = A]$$

$$= 1\,NA^{-1}\,m^{-1} = \mathbf{1\ tesla.}$$

Thus the SI unit of magnetic field is ***tesla*** (T).

One tesla is that magnetic field in which a charge of 1 C moving with a velocity of 1 ms $^{-1}$ at right angles to the field experiences a force of one newton.

A field of one tesla is a very strong magnetic field. Very often the magnetic fields are expressed in terms of a smaller unit, called the gauss (G).

$$1 \text{ gauss} = 10^{-4} \text{tesla}$$

Table 4.1 Some Typical Magnetic Fields

Surface of a neutron star	10^8 T
Large field in the laboratory	1 T
Field near a bar magnet	10^{-2} T
Field on the earth's surface	10^{-4} T
Field in interstellar space	10^{-12} T

Dimensions of magnetic field. Clearly,

$$[B] = \frac{[F]}{[q][v][\sin\theta]} = \frac{MLT^{-2}}{AT\,.\,LT^{-1}\,.1}$$

$$= [\mathbf{MT^{-2}A^{-1}}].$$

Here A represents current.

4.12 LORENTZ FORCE

13. *What is Lorentz force ? Write an expression for it.*

Lorentz force. *The total force experienced by a charged particle moving in a region where both electric and magnetic fields are present, is called Lorentz force.*

A charge q in an electric field $\vec{E}$ experiences the *electric force,*

$$\vec{F}_e = q\vec{E}$$

This force acts in the direction of field $\vec{E}$ and is independent of the velocity of the charge.

The *magnetic force* experienced by the charge q moving with velocity $\vec{v}$ in the magnetic field $\vec{B}$ is given by

$$\vec{F}_m = q(\vec{v} \times \vec{B})$$

This force acts perpendicular to the plane of $\vec{v}$ and $\vec{B}$ and depends on the velocity $\vec{v}$ of the charge.

The total force, or the *Lorentz force*, experienced by the charge q due to both electric and magnetic field is given by

$$\vec{F} = \vec{F}_e + \vec{F}_m$$

or

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

For Your Knowledge

- A static charge is a source of electric field only while a moving charge is source of both electric and magnetic fields.
- A moving charge produces a magnetic field which, in turn, exerts a force on another moving charge.
- A stationary source does not produce any magnetic field to interact with an external magnetic field. Hence no force is exerted on stationary charge in a magnetic field.
- An electric charge always experiences a force in an electric field, whether the charge is stationary or in motion.
- A charge moving parallel or antiparallel to the direction of the magnetic field does not experience any magnetic Lorentz force.
- If in a field, the force experienced by a moving charge depends on the strength of the field and not on the velocity of the charge, then the field must be an electric field.
- If in a field, the force experienced by a moving charge depends not only on the strength of the field but also on the velocity of the charge, then the field must be a magnetic field.

Examples Based on Force on Moving Charges in a Magnetic Field

Formulae Used

Force on a charge q moving with velocity v in a magnetic field at an angle θ with it is

$$F = qvB\sin\theta$$

The direction of the force is given by Fleming's left hand rule.

Units Used

Force F is in newton, charge q in coulomb, velocity v in ms^{-1} and B in tesla.

Example 35. *A proton enters a magnetic field of flux density 2.5 T with a velocity of $1.5 \times 10^7\ ms^{-1}$ at an angle of 30° with the field. Find the force on the proton.*

Solution. Here $q = e = 1.6 \times 10^{-19}$ C,

$$v = 1.5 \times 10^7\ \text{ms}^{-1},\quad B = 2.5\ \text{T},\ \theta = 30°$$

Force, $F = qvB \sin\theta$

$$= 1.6 \times 10^{-19} \times 1.5 \times 10^7 \times 2.5 \times \sin 30°$$

$$= \mathbf{3 \times 10^{-12}\ N}.$$

Example 36. *An alpha particle is projected vertically upward with a speed of 3×10^4 km s^{-1} in a region where a magnetic field of magnitude 1.0 T exists in the direction south to north. Find that magnetic force that acts on the particle.*

Solution. Charge on α-particle,

$$q = +2e = 2 \times 1.6 \times 10^{-19}\ \text{C}$$

Here $v = 3 \times 10^4$ km s^{-1} $= 3 \times 10^7$ ms^{-1}, $B = 1.0$ T, $\theta = 90°$.

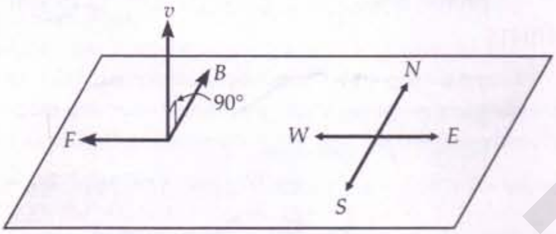


Fig. 4.59

Magnetic force on the α-particle is

$$F = qvB \sin\theta$$

$$= 2 \times 1.6 \times 10^{-19} \times 3 \times 10^7 \times 1.0 \times \sin 90°$$

$$= \mathbf{9.6 \times 10^{-12}\ N}$$

According to Fleming's left hand rule, the magnetic force on the α-particle acts towards *west*.

Example 37. *An electron is moving northwards with a velocity of 3.0×10^7 ms^{-1} in a uniform magnetic field of 10 T directed eastwards. Find the magnitude and the direction of the force on the electron.*

Solution. $q = e = 1.6 \times 10^{-19}$ C, $v = 3.0 \times 10^7$ ms^{-1}, $B = 10$ T, $\theta = 90°$.

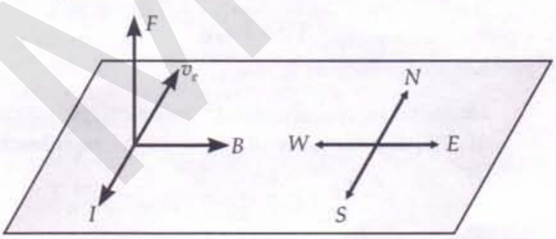


Fig. 4.60

The magnitude of magnetic force on the electron is

$$F = qvB \sin\theta = 1.6 \times 10^{-19} \times 3 \times 10^7 \times 10 \times \sin 90°$$

$$= \mathbf{4.8 \times 10^{-11}\ N}$$

As the electron moves northwards, direction of current is eastwards. According to Fleming's left hand rule, the magnetic force on the electron acts *vertically upwards*.

Example 38. *A positive charge of 1.5 μC is moving with a speed of 2×10^6 ms^{-1} along the positive X-axis. A magnetic field, $\vec{B} = (0.2\,\hat{j} + 0.4\,\hat{k})$ tesla acts in space. Find the magnetic force acting on the charge.*

Solution. Here $q = 1.5\ \mu\text{C} = 1.5 \times 10^{-6}$ C,

$$\vec{v} = 2 \times 10^6\,\hat{i}\ \text{ms}^{-1},\quad \vec{B} = (0.2\,\hat{j} + 0.4\,\hat{k})\ \text{T}$$

Magnetic force on the positive charge is

$$\vec{F} = q(\vec{v} \times \vec{B})$$

$$= 1.5 \times 10^{-6}[2 \times 10^6\,\hat{i} \times (0.2\,\hat{j} + 0.4\,\hat{k})]$$

$$= 3.0\,[0.2\,\hat{i} \times \hat{j} + 0.4\,\hat{i} \times \hat{k}]$$

$$= (0.6\,\hat{k} - 1.2\,\hat{j})\ \text{N}. \quad [\because \hat{i} \times \hat{j} = \hat{k},\ \hat{i} \times \hat{k} = -\hat{j}]$$

Example 39. *A 5.0 MeV proton is falling vertically downward through a region of magnetic field 1.5 T acting horizontally from south to north. Find the magnitude and the direction of the magnetic force exerted on the proton. Take mass of the proton as 1.6×10^{-27} kg.*

Solution. Kinetic energy of the proton is

$$\frac{1}{2}mv^2 = 5.0\ \text{MeV} = 5 \times 1.6 \times 10^{-13}\ \text{J}$$

or
$$v^2 = \frac{2 \times 5 \times 1.6 \times 10^{-13}}{m}\ \text{J}$$

$$= \frac{10 \times 1.6 \times 10^{-13}}{1.6 \times 10^{-27}} = 10 \times 10^{14}$$

$$\therefore \quad v = 3.16 \times 10^7\ \text{m s}^{-1}$$

Force on the proton is

$$F = qvB \sin 90°$$

$$= 1.6 \times 10^{-19} \times 3.16 \times 10^7 \times 1.5 \times 1$$

$$= \mathbf{7.58 \times 10^{-12}\ N}$$

According to Fleming's left hand rule, the magnetic force on the proton acts eastwards.

Example 40. *A long straight wire AB carries a current of 4 A. A proton P travels at 4×10^6 m/s, parallel to the wire, 0.2 m from it and in a direction opposite to the current as shown in Fig. 4.61. Calculate the force which the magnetic field of current exerts on the proton. Also specify the direction of the force.*

[CBSE OD 02]

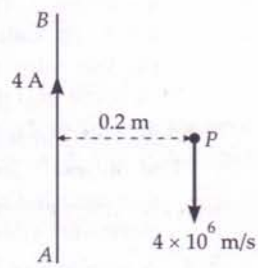


Fig. 4.61

Solution. Magnetic field at point P due to the current in wire AB,

$$B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 4}{2\pi \times 0.2} = 4 \times 10^{-6} \text{ T}$$

This field acts on the proton normally into the plane of paper. According to Fleming's left hand rule, a magnetic force acts on the proton towards right in the plane of paper. The magnitude of this force is

$$F = qvB \sin 90°$$
$$= 1.6 \times 10^{-19} \times 4 \times 10^{6} \times 4 \times 10^{-6} \times 1$$
$$= \mathbf{2.56 \times 10^{-18}\ N}.$$

Example 41. *Copper has 8.0×10^{28} electrons per cubic metre. A copper wire of length 1 m and cross-sectional area 8.0×10^{-6} m² carrying a current and lying at right angle to a magnetic field of strength 5×10^{-3} T experiences a force of 8.0×10^{-2} N. Calculate the drift velocity of free electrons in the wire.*

Solution. $n = 8 \times 10^{28} \text{m}^{-3}, l = 1$ m

$$A = 8 \times 10^{-6} \text{m}^2, \quad e = 1.6 \times 10^{-19} \text{ C}$$

Total charge contained in the wire,

$$q = \text{Volume of wire} \times ne = Alne$$
$$= 8 \times 10^{-6} \times 1 \times 8 \times 10^{28} \times 1.6 \times 10^{-19} \text{ C}$$
$$= 102.4 \times 10^{3} \text{ C}$$

If v_d is the drift speed of electrons, then

$$F = q\, v_d\, B \sin 90° = q\, v_d\, B$$

$$\therefore\ v_d = \frac{F}{qB} = \frac{8.0 \times 10^{-2}}{102.4 \times 10^{3} \times 5 \times 10^{-3}} \text{ ms}^{-1}$$

$$= \mathbf{1.56 \times 10^{-4}\ ms^{-1}}.$$

Problems For Practice

1. An electron moving with a velocity of $5.0 \times 10^{7} \text{ms}^{-1}$ enters a magnetic field of 1.0 Wb m^{-2} at an angle of 30°. Calculate the force on the electron.
(Ans. 4.0×10^{-12} N)

2. An α-particle of mass 6.65×10^{-27} kg and charge twice that of an electron but of positive sign travels at right angles to a magnetic field with a speed of $6 \times 10^{5} \text{ms}^{-1}$. The strength of the magnetic field is 0.2 T. (*i*) Calculate the force on the α-particle. (*ii*) Also calculate its acceleration.
[Ans. (*i*) 3.84×10^{-14} N (*ii*) 5.77×10^{12} ms^{-2}]

3. An electron is moving northwards with a velocity of 10^{7} ms^{-1} in a magnetic field of 3 T, directed downwards. Calculate the instantaneous force on the electron. (Ans. 4.8×10^{-12} N, vertically upwards)

4. A solenoid, of length 1.5 m, has a radius of 1.5 cm and has a total of 1500 turns wound on it. It carries a current of 3 A. Calculate the magnitude of the axial magnetic field inside the solenoid. If an electron were to move with a speed of 2×10^{4} ms^{-1} along the axis of this current carrying solenoid, what would be the force experienced by this electron ?
[CBSE D 08C] (Ans. 0.38 T, 0)

5. An electron is moving at 10^{6} ms^{-1} in a direction parallel to a current of 5 A, flowing through an infinitely long straight wire, separated by a perpendicular distance of 10 cm in air. Calculate the magnitude of the force experienced by the electron.
[CBSE D 99] (Ans. 1.6×10^{-18} N)

6. A proton of energy 3.4 MeV moves vertically downwards through a horizontal magnetic field of 3 T which acts from south to north. What is the force on the proton ? Mass of proton is 1.7×10^{-27} kg ; charge on proton is 1.6×10^{-19} C. (Ans. 12.15×10^{-12} N)

HINTS

1. $q = e = 1.6 \times 10^{-19}$ C, $v = 5.0 \times 10^{7}$ ms^{-1}
$B = 1.0$ Wb m^{-2}, $\theta = 30°$
Force, $F = qvB \sin \theta$
$= 1.6 \times 10^{-19} \times 5.0 \times 10^{7} \times 1.0 \times \sin 30°$
$= \mathbf{4.0 \times 10^{-12}\ N}.$

2. (*i*) Here $m = 6.65 \times 10^{27}$ kg,
$q = +2e = 2 \times 1.6 \times 10^{-16}$C, $B = 0.2$ T,
$v = 6 \times 10^{5}$ ms^{-1}, $\theta = 90°$
$F = qvB \sin 90°$
$= 2 \times 1.6 \times 10^{-19} \times 6 \times 10^{5} \times 0.2 \times 1$ N
$= \mathbf{3.84 \times 10^{-14}\ N}$
$$a = \frac{F}{m} = \frac{3.84 \times 10^{-14}}{6.65 \times 10^{-27}} = \mathbf{5.77 \times 10^{12}\ ms^{-2}}.$$

3. $F = qvB \sin 90° = 1.6 \times 10^{-19} \times 10^{7} \times 3 \times 1$
$= \mathbf{4.8 \times 10^{-12}\ N}$
According to Fleming's left hand rule, the force acts vertically upwards.

4. $$B = \frac{\mu_0 NI}{l} = \frac{4\pi \times 10^{-7} \times 1500 \times 3}{1.5 \times 10^{-2}} \text{ T} = \mathbf{0.38\ T}$$
Force, $F = evB \sin 0° = \mathbf{0}$.

5. Magnetic field of the straight wire carrying a current of 2 A, at a distance of 10 cm or 0.1 m from it is
$$B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 5}{2\pi \times 0.1} = 10^{-5} \text{ T}$$
This field acts perpendicular to the direction of the electron. So magnetic force on the electron is
$F = q\, v\, B \sin 90°$
$= 1.6 \times 10^{-19} \times 10^{6} \times 10^{-5} \times 1 = \mathbf{1.6 \times 10^{-18}\ N}.$

6. Proceed as in Example 39, on page 4.31.

4.13 WORK DONE BY A MAGNETIC FORCE ON A CHARGED PARTICLE IS ZERO

14. *Show that the work done by a magnetic field on a moving charged particle is always zero.*

Work done by a magnetic force on a charged particle. The magnetic force $\vec{F} = q(\vec{v} \times \vec{B})$ always acts perpendicular to the velocity $\vec{v}$ or the direction of motion of charge q. Therefore,

$$\vec{F} \cdot \vec{v} = q(\vec{v} \times \vec{B}) \cdot \vec{v} = 0$$

According to Newton's second law,

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$$

$$\therefore \quad m\frac{d\vec{v}}{dt} \cdot \vec{v} = 0$$

or $$\frac{m}{2}\left[\frac{d\vec{v}}{dt} \cdot \vec{v} + \vec{v} \cdot \frac{d\vec{v}}{dt}\right] = 0$$

or $$\frac{m}{2}\frac{d}{dt}(\vec{v} \cdot \vec{v}) = 0$$

or $$\frac{d}{dt}\left(\frac{1}{2}mv^2\right) = 0 \qquad [\vec{v} \cdot \vec{v} = v^2]$$

or $$\frac{dK}{dt} = 0$$

or $$K = \text{constant}$$

Thus *a magnetic force does not change the kinetic energy of the charged particle.* This indicates that the *speed of the particle does not change.* According to the work-energy theorem, the change in kinetic energy is equal to the work done on the particle by the net force. Hence the *work done* on the charged particle *by the magnetic force is zero.*

4.14 MOTION OF A CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD

15. *Discuss the motion of a charged particle in a uniform magnetic field with initial velocity (i) parallel to the field, (ii) perpendicular to the magnetic field and (iii) at an arbitrary angle with the field direction.*

Motion of a charged particle in a uniform magnetic field. When a charged particle having charge q and velocity $\vec{v}$ enters a magnetic field $\vec{B}$, it experiences a force

$$\vec{F} = q(\vec{v} \times \vec{B})$$

The direction of this force is perpendicular to both $\vec{v}$ and $\vec{B}$. The magnitude of this force is

$$F = qv\,B\sin\theta$$

Following *three* cases are possible :

1. ***When the initial velocity is parallel to the magnetic field.*** Here $\theta = 0°$, so $F = qvB\sin 0° = 0$.

Thus the parallel magnetic field does not exert any force on the moving charged particle. The charged particle will continue to move along the line of force.

2. ***When the initial velocity is perpendicular to the magnetic field.*** Here $\theta = 90°$, so $F = qvB\sin 90° = qvB$ = a maximum force. As the magnetic force acts on a particle perpendicular to its velocity, it does not do any work on the particle. It does not change the kinetic energy or speed of the particle.

Figure 4.62 shows a magnetic field $\vec{B}$ directed normally into the plane of paper, as shown by small crosses. A charge $+q$ is projected with a speed v in the plane of the paper. The velocity is perpendicular to the

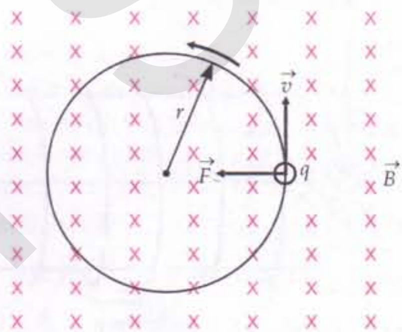


Fig. 4.62 A positively charged particle moving in a magnetic field directed into the plane of paper.

magnetic field. A force $F = qvB$ acts on the particle perpendicular to both $\vec{v}$ and $\vec{B}$. This force continuously deflects the particle sideways without changing its speed and the particle will move along a circle perpendicular to the field. Thus the magnetic force provides the centripetal force. Let r be the radius of the circular path. Now

$$\text{Centripetal force, } \frac{mv^2}{r} = \text{Magnetic force, } qvB$$

or $$r = \frac{mv}{qB}$$

Thus the radius of the circular orbit is inversely proportional to the specific charge (charge to mass ratio q/m) and to the magnetic field.

$$\text{Period of revolution} = \frac{\text{Circumference}}{\text{Speed}}$$

or $$T = \frac{2\pi r}{v} = \frac{2\pi}{v} \cdot \frac{mv}{qB} = \frac{2\pi m}{qB}$$

Clearly, the time period is independent of v and r. If the particle moves faster, the radius is larger, it has to move along a larger circle so that the time taken is the same.

The frequency of revolution is

$$f_c = \frac{1}{T} = \frac{qB}{2\pi m}$$

This frequency is called ***cyclotron frequency***.

3. ***When the initial velocity makes an arbitrary angle with the field direction.*** A uniform magnetic field $\vec{B}$ is set up along +ve X-axis. A particle of charge q and mass m enters the field $\vec{B}$ with velocity $\vec{v}$ inclined at angle θ with the direction of the field $\vec{B}$, as shown in Fig. 4.63.

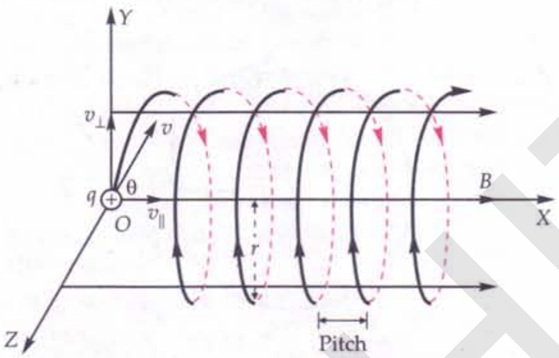


Fig. 4.63 Helical motion of charged particle in a magnetic field.

The velocity $\vec{v}$ can be resolved into two rectangular components :

1. The component $v_\parallel$ along the direction of the field *i.e.*, along X-axis. Clearly

$$v_\parallel = v \cos\theta$$

The parallel component remains unaffected by the magnetic field and so the charged particle continues to move along the field with a speed of $v \cos\theta$.

2. The component $v_\perp$ perpendicular to the direction of the field *i.e.*, in the YZ-plane. Clearly

$$v_\perp = v \sin\theta$$

Due to this component of velocity, the charged particle experiences a force $F = qv_\perp B$ which acts perpendicular to both $v_\perp$ and $\vec{B}$. This force makes the particle move along a circular path in the YZ-plane. The radius of the circular path is

$$r = \frac{mv_\perp}{qB} = \frac{mv \sin\theta}{qB}$$

The period of revolution is

$$T = \frac{2\pi r}{v_\perp} = \frac{2\pi}{v \sin\theta} \cdot \frac{mv \sin\theta}{qB} = \frac{2\pi m}{qB}$$

Thus a charged particle moving in a uniform magnetic field has two concurrent motions : a linear motion in the direction of $\vec{B}$ (along X-axis) and a circular motion in a plane perpendicular to $\vec{B}$ (in YZ-plane). Hence the resultant path of the charged particle will be a helix, with its axis along the direction of $\vec{B}$.

The linear distance travelled by the charged particle in the direction of the magnetic field during its period of revolution is called pitch of the helical path.

$$\text{pitch} = v_\parallel \times T = v \cos\theta \times \frac{2\pi m}{qB} = \frac{2\pi mv \cos\theta}{qB}.$$

4.15 MOTION OF A CHARGE IN PERPENDICULAR MAGNETIC AND ELECTRIC FIELDS

16. *Electric and magnetic fields are applied mutually perpendicular to each other. Show that a charged particle will follow a straight line path perpendicular to both of these fields, if its velocity is E/B in magnitude.*

Velocity selector. Suppose a beam of charged particles, say electrons, possessing a range of speeds passes through a slit S_1 and then enters a region in which crossed (perpendicular) electric and magnetic fields exist. As shown in Fig. 4.64, the electric field $\vec{E}$ acts in the downward direction and deflects the electrons in the upward direction. The magnetic field $\vec{B}$ acts normally into the plane of paper and deflects the electrons in the downward direction.

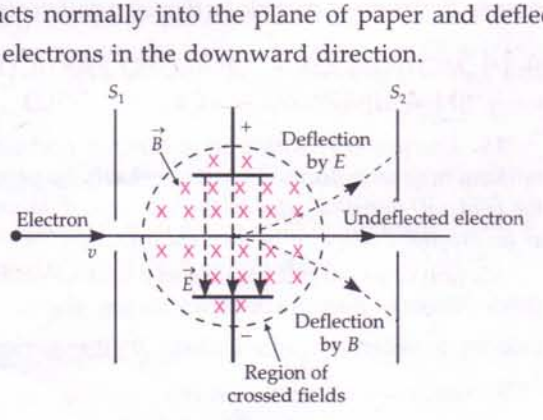


Fig. 4.64 Motion of an electron in a region of crossed magnetic and electric fields.

Only those electrons will pass undeflected through the slit S_2 on which the electric and magnetic forces are equal and opposite. The velocity v of the undeflected electrons is given by

$$eE = evB \quad \text{or} \quad v = \frac{E}{B}$$

Such an arrangement can be used to *select* charged particles of a particular velocity out of a beam in which the particles are moving with different speeds. This arrangement is called ***velocity selector*** or ***velocity filter***. This method was used by *J.J. Thomson* to determine the charge to mass ratio (e/m) of an electron.

Examples based on Motion of Charges in Electric and Magnetic Fields

Formulae Used

1. Electric force on a charge, $F_e = qE$
2. Magnetic force on a charge, $F_m = qvB\sin\theta$
3. In a perpendicular magnetic field, the charge follows a circular path.

$$qvB = \frac{mv^2}{r} \quad \text{or} \quad r = \frac{mv}{qB}$$

$$T = \frac{2\pi m}{qB} \quad \text{and} \quad f = \frac{qB}{2\pi m}$$

4. When $\vec{v}$ makes angle θ with $\vec{B}$, the charge follows helical path.

$$r = \frac{mv_\perp}{qB} = \frac{mv\sin\theta}{qB}; \; T = \frac{2\pi r}{v_\perp} = \frac{2\pi m}{qB}$$

Pitch of helix, $h = v_\parallel T = v\cos\theta \,.\, T = \frac{2\pi mv\cos\theta}{qB}$

5. K.E. gained by an electron when accelerated through a potential difference V,

$$\frac{1}{2}mv^2 = eV \quad \therefore \quad v = \sqrt{\frac{2eV}{m}}$$

Units Used

E is in Vm^{-1} or NC^{-1}, B in tesla, v in ms^{-1}, r in metre.

Example 42. *An electron moving horizontally with a velocity of 4×10^4 m/s enters a region of uniform magnetic field of 10^{-5} T acting vertically downward as shown in Fig. 4.65(a). Draw its trajectory and find out the time it takes to come out of the region of magnetic field.* [CBSE F 15]

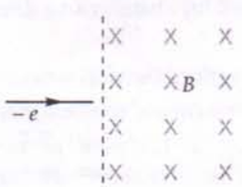


Fig. 4.65 (*a*)

Solution. The electron moves along semicircular trajectory inside the magnetic field and comes out, as shown in Fig. 4.65(*b*). Radius r of the path is given by

$$\frac{mv^2}{r} = qvB$$

or
$$r = \frac{mv}{qB} = \frac{9.1 \times 10^{-31} \times 4 \times 10^4}{1.6 \times 10^{-19} \times 10^{-5}} \text{ m}$$

$$= \frac{9.1 \times 4}{1.6} \times 10^{-3} \text{ m} = 22.75 \times 10^{-3} \text{ m}$$

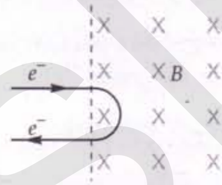


Fig. 4.65 (*b*)

Time taken to come out of the region of magnetic field,

$$t = \frac{\pi r}{v} = \frac{22 \times 22.75 \times 10^{-3}}{7 \times 4 \times 10^4} \text{ s}$$

$$= 17.875 \times 10^{-7} \text{ s} \simeq \mathbf{1.8 \times 10^{-6} s.}$$

Example 43. *An electron travels in a circular path of radius 20 cm in a magnetic field 2×10^{-3} T. (i) Calculate the speed of the electron. (ii) What is the potential difference through which the electron must be accelerated to acquire this speed ?*

Solution. Here $r = 20 \text{ cm} = 20 \times 10^{-2}$ m,

$B = 2 \times 10^{-3}$ T, $e = 1.6 \times 10^{-19}$ C, $m = 9.1 \times 10^{-31}$ kg

(*i*) Magnetic force on the electron

= Centripetal force on electron

$$evB = \frac{mv^2}{r}$$

$$\therefore \text{ Speed, } \quad v = \frac{eBr}{m}$$

$$= \frac{1.6 \times 10^{-19} \times 2 \times 10^{-3} \times 20 \times 10^{-2}}{9.1 \times 10^{-31}}$$

$$= \mathbf{7.0 \times 10^7 \; ms^{-1}}.$$

(*ii*) If V is the p.d. required to give speed v to the electron, then

$$eV = \frac{1}{2}mv^2$$

or
$$V = \frac{mv^2}{2e} = \frac{9.1 \times 10^{-31} \times (7.0 \times 10^7)^2}{2 \times 1.6 \times 10^{-19}}$$

$$= 13.9 \times 10^3 \text{ V} \simeq \mathbf{14 \; kV}.$$

Example 44. *An electron after being accelerated through a potential difference of 10^4 V enters a uniform magnetic field of 0.04 T perpendicular to its direction of motion. Calculate the radius of curvature of its trajectory.*

Solution. Here $V = 10^4$ V, $B = 0.04$ T,

$e = 1.6 \times 10^{-19}$ C, $m = 9.1 \times 10^{-31}$ kg

An electron accelerated through a p.d. V acquires a velocity v given by

$$\frac{1}{2}mv^2 = eV \quad \text{or} \quad v = \sqrt{\frac{2eV}{m}}$$

As the electron describes a circular path of radius of r in the perpendicular magnetic field B, therefore,

$$\frac{mv^2}{r} = evB$$

or $$r = \frac{mv}{eB} = \frac{m}{eB}\sqrt{\frac{2eV}{m}} = \frac{\sqrt{2meV}}{eB}$$

$$= \frac{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 10^4}}{1.6 \times 10^{-19} \times 0.04}$$

$$= \frac{5.4 \times 10^{-23}}{1.6 \times 10^{-19} \times 0.04}$$

$$= 8.43 \times 10^{-3}\,\text{m} = \mathbf{8.43\ mm.}$$

Example 45. *If a particle of charge q is moving with velocity v along the z-axis and the magnetic field B is acting along the x-axis, use the expression $\vec{F} = q(\vec{v} \times \vec{B})$ to find the direction of the force F acting on it.*

A beam of proton passes undeflected with a horizontal velocity v, through a region of electric and magnetic fields, mutually perpendicular to each other and normal to the direction of the beam. If the magnitudes of the electric and magnetic fields are 100 kV/m and 50 mT respectively, calculate : (i) velocity v of the beam. (ii) force with which it strikes a target on a screen, if the proton beam current is equal to 0.80 mA. [CBSE OD 08]

Solution. $\vec{F} = q(\vec{v} \times \vec{B}) = q(v\hat{j} \times B\hat{k})$

$$= qvB\hat{j} \times \hat{k} = qvB\hat{i}$$

Thus the force F acts on the charge q along the +ve x-direction.

(*i*) For undeflected proton beam,

$$qvB = qE$$

$$v = \frac{E}{B} = \frac{100\ \text{kVm}^{-1}}{50\ \text{mT}} = \frac{100 \times 10^3\ \text{Vm}^{-1}}{50 \times 10^{-3}\text{T}}$$

$$= \mathbf{2 \times 10^6\ ms^{-1}}.$$

(*ii*) Current carried by proton beam,

$$I = 0.8\ \text{mA} = 8 \times 10^{-4}\ \text{A}$$

Number of protons striking the screen per second,

$$n = \frac{I}{e} = \frac{8 \times 10^{-4}}{1.6 \times 10^{-19}} = 5 \times 10^{15}\,\text{s}^{-1}$$

$$m_p = 1.675 \times 10^{-27}\,\text{kg}$$

Force with which a proton beam strikes a target on the screen,

$$F = \frac{dp}{dt} = m_p nv$$

$$= 1.675 \times 10^{-27} \times 5 \times 10^{15} \times 2 \times 10^6\ \text{N}$$

$$= \mathbf{1.675 \times 10^{-5}\ N.}$$

Example 46. *An electron beam passes through a magnetic field of 2×10^{-3} Wb m^{-2} and an electric field of 3.4×10^4 Vm^{-1}, both acting simultaneously. If the path of the electron remains undeviated, calculate the speed of the electrons. If the electric field is removed, what will be the radius of the circular path ? Mass of an electron $= 9.1 \times 10^{-31}$ kg.*

Solution. Here $B = 2 \times 10^{-3}$ Wb m^{-2},

$E = 3.4 \times 10^4$ Vm^{-1}

Magnetic force on the electron

= Electric force on the electron

or $evB = eE$

∴ Velocity of electrons,

$$v = \frac{E}{B} = \frac{3.4 \times 10^4}{2 \times 10^{-3}}\ \text{ms}^{-1} = 1.7 \times 10^7\ \text{ms}^{-1}$$

When electric field has been removed,

Force exerted by the magnetic field on an electron

= Centripetal force on an electron

i.e., $$evB = \frac{mv^2}{R}$$

or $$R = \frac{mv}{eB} = \frac{9.1 \times 10^{-31} \times 1.7 \times 10^7}{1.6 \times 10^{-19} \times 2 \times 10^{-3}}$$

$$= 4.8 \times 10^{-2}\ \text{m} = \mathbf{4.8\ cm}.$$

Example 47. *In a chamber a uniform magnetic field of 8.0 G (1 G $= 10^{-4}$ T) is maintained. An electron with a speed of 4.0×10^6 ms^{-1} enters the chamber in a direction normal to the field.*

(*i*) *Describe the path of the electron.*

(*ii*) *What is the frequency of revolution of the electron ?*

(*iii*) *What happens to the path of the electron if it progressively loses its energy due to collisions with the atoms or molecules of the environment ?*

[NCERT]

Solution. (*i*) The path of the electron is a circle of radius r given by

$$r = \frac{mv}{eB}$$

Here $B = 8.0\ \text{G} = 8.0 \times 10^{-4}\,\text{T}$, $v = 4.0 \times 10^{6}\ \text{ms}^{-1}$, $e = 1.6 \times 10^{-19}$ C, $m = 9.1 \times 10^{-31}$ kg

$$\therefore \quad r = \frac{9.1 \times 10^{-31} \times 4.0 \times 10^{6}}{1.6 \times 10^{-19} \times 8.0 \times 10^{-4}}$$

$$= 2.8 \times 10^{-2}\ \text{m} = \mathbf{2.8\ cm.}$$

The sense of rotation of the electron in its orbit can be ascertained from the direction of the centripetal force $\vec{F} = -e(\vec{v} \times \vec{B})$. Thus if we look *along* the direction of $\vec{B}$, the electron revolves clockwise.

(*b*) The frequency of revolution of the electron in its circular orbit is

$$f = \frac{eB}{2\pi m} = \frac{1.6 \times 10^{-19} \times 8.0 \times 10^{-4}}{2\pi \times 9.1 \times 10^{-31}}$$

$$= 0.22 \times 10^{8}\ \textbf{Hz} = \textbf{22 MHz.}$$

(*c*) In successive collisions, electron loses its speed progressively. If after collision its velocity vector remains in the same plane of the initial circular orbit, the radius of the circular orbit will decrease in proportion to the decreasing speed. Otherwise, the path of the electron will be helical between two collisions.

Example 48. *A monoenergetic electron beam of initial energy 18 keV moving horizontally is subjected to a horizontal magnetic field of 0.4 G normal to its initial direction. Calculate the vertical deflection of the beam over a distance of 30 cm.* [CBSE Sample Paper 98]

Solution. Under the action of the magnetic field, the electrons will move along a circular path.

$\therefore$ Centripetal force on an electron

= Magnetic force on an electron

$$\frac{mv^2}{r} = evB$$

or
$$r = \frac{mv}{eB} = \frac{\sqrt{2m.1/2\ mv^2}}{eB}$$

Here $m = 9.11 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C, $B = 0.40\ \text{G} = 0.40 \times 10^{-4}$ T

$$\text{K.E.} = \frac{1}{2} mv^2 = 18\ \text{keV} = 18 \times 1.6 \times 10^{-16}\ \text{J}$$

$$\therefore \quad r = \frac{\sqrt{2 \times 9.11 \times 10^{-31} \times 18 \times 1.6 \times 10^{-16}}}{1.6 \times 10^{-19} \times 0.4 \times 10^{-4}}$$

$$= 11.32\ \text{m}$$

Thus the electron moves in a circle of radius 11.32 m, as shown in Fig. 4.66. As it covers a distance $PQ = 30$ cm, it goes down through a vertical distance

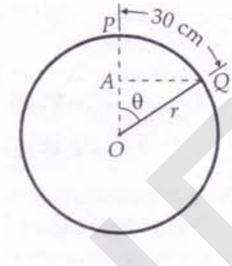


Fig. 4.66

equal to PA. If θ is the angle subtended by arc PQ at the centre O, then

$$PA = OP - OA = r - r\cos\theta = r(1 - \cos\theta)$$

Now
$$\theta = \frac{\text{Arc}}{\text{Radius}} = \frac{30 \times 10^{-2}}{11.32} = 0.02650\ \text{rad}$$

$$= \frac{0.02650 \times 180}{\pi} = 1.52^\circ$$

$$\therefore \quad \cos\theta = \cos 1.52^\circ = 0.99965$$

Hence
$$PA = 11.32\,(1 - 0.99965)$$

$$= 3.9744 \times 10^{-3}\ \text{m} \simeq \mathbf{4\ mm.}$$

Example 49. *A beam of protons enters a uniform magnetic field of 0.3 T with a velocity of 4×10^{5} ms^{-1} at an angle of 60° to the field. Find the radius of the helical path taken by the beam. Also find the pitch of the helix (distance travelled by a proton parallel to the magnetic field during one period of rotation). Mass of proton is 1.67×10^{-27} kg.* [IIT 86]

Solution. The components of the proton's velocity parallel and perpendicular to the magnetic field are

$$v_{\parallel} = v\cos 60^\circ = 4 \times 10^{5} \times \frac{1}{2} = 2 \times 10^{5}\ \text{ms}^{-1}$$

$$v_{\perp} = v\sin 60^\circ = 4 \times 10^{5} \times \frac{\sqrt{3}}{2} = 3.464 \times 10^{5}\ \text{ms}^{-1}$$

The component $v_{\parallel}$ makes the electron move along the field B while $v_{\perp}$ makes the proton move along a circular path. Hence the path of the proton is a helix. The radius r of the helix is given by

$$q v_{\perp} B = \frac{m v_{\perp}^2}{r}$$

or
$$r = \frac{mv_{\perp}}{qB} = \frac{1.67 \times 10^{-27} \times 3.464 \times 10^{5}}{1.6 \times 10^{-19} \times 0.3}$$

$$= 12 \times 10^{-3}\ \text{m} = \mathbf{1.2\ cm.}$$

Period of revolution of the electron is

$$T=\frac{2\pi r}{v_{\perp}}=\frac{2\times 3.14\times 12\times 10^{-3}}{3.464\times 10^{5}}$$

$$=21.75\times 10^{-8}\,\text{s}$$

Pitch of the helix is

$$p=v_{\parallel}\times T=2\times 10^{5}\times 21.75\times 10^{-8}$$

$$=43.5\times 10^{-3}\,\text{m}=\mathbf{4.35\ cm.}$$

Example 50. *A proton projected in a magnetic field of 0.02 T travels along a helical path of radius 5.0 cm and pitch 20 cm. Find the components of the velocity of the proton along and perpendicular to the magnetic field. Take the mass of the proton $=1.6\times 10^{-27}$ kg.*

Solution. Radius of helical path, $r=\frac{mv_{\perp}}{qB}$

$$\therefore\ v_{\perp}=\frac{rqB}{m}=\frac{5\times 10^{-2}\times 1.6\times 10^{-19}\times 0.02}{1.6\times 10^{-27}}$$

$$=\mathbf{1.0\times 10^{5}\ ms^{-1}}$$

Period of revolution,

$$T=\frac{2\pi r}{v_{\perp}}=\frac{2\pi\times 5\times 10^{-2}}{1.0\times 10^{5}}=\pi\times 10^{-6}\ \text{s}$$

$$v_{\parallel}=\frac{\text{Pitch}}{T}=\frac{20\times 10^{-2}}{\pi\times 10^{-6}}=\mathbf{6.37\times 10^{4}\ ms^{-1}}.$$

Problems For Practice

1. An electron entering a magnetic field of 10^{-2} T with a velocity of 10^{7} ms^{-1} describes a circle of radius 6×10^{-3} m. Calculate e/m of the electron. (Ans. 1.67×10^{11} C kg^{-1})
2. An electron after being accelerated through a potential difference of 100 V enters a uniform magnetic field of 0.004 T perpendicular to its direction of motion. Calculate the radius of the path described by the electron. **[CBSE OD 92]** (Ans. 8.4 mm)
3. A particle having a charge of 100 μC and a mass of 10 mg is projected in a uniform magnetic field of 25 mT with a speed of 10 ms^{-1} in a direction perpendicular to the field. What will be the period of revolution of the particle in the magnetic field ? (Ans. 25 s)
4. An electron having a kinetic energy of 100 eV circulates in a path of radius 10 cm in a magnetic field. Find the magnetic field and the number of revolutions made by the electron per second. (Ans. 3.4×10^{-4} T, 9.4×10^{6} rps)
5. An electron beam passes through a magnetic field of 2×10^{-3} Wb m^{-2} and an electric field of 1.0×10^{4} V m^{-1}, both acting simultaneously. If the path of the electrons remains undeviated, calculate the speed of the electrons. If the electric field is removed, what will be the radius of the circular path ? (Ans. 5×10^{6} ms^{-1}, 1.43 cm)
6. An electron moving perpendicular to a uniform magnetic field completes a circular orbit in 10^{-6} s. Calculate the value of the magnetic field. Mass of electron $=9\times 10^{-31}$ kg. (Ans. 3.5×10^{-3} T)
7. Find the flux density of the magnetic field to cause 62.5 eV electron to move in a circular path of radius 5 cm. Given $m_e=9.1\times 10^{-31}$ kg and $e=1.6\times 10^{-19}$ C. (Ans. 5.335×10^{-4} T)
8. An electron of energy 2000 eV describes a circular path in a magnetic field of 0.2 T. What is the radius of path ? Take $m_e=9\times 10^{-31}$ kg, $e=1.6\times 10^{-19}$ C. (Ans. 0.75 mm)
9. What should be the minimum magnitude and direction of the magnetic field that must be produced at the equator of earth so that a proton may go round the earth with a speed of 1.0×10^{7} ms^{-1} ? Earth's radius is 6.4×10^{6} m. (Ans. 1.63×10^{-8} T, perpendicular to the equator in a horizontal direction)
10. A stream of charged particles possessing a range of speeds enters region I after passing through a slit S_1 (Fig. 4.67). In region I there exist crossed (perpen-

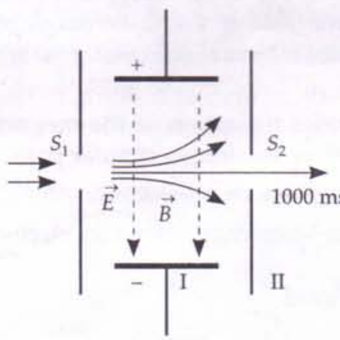


Fig. 4.67

dicular) electric and magnetic fields. The electric field has magnitude 100 Vm^{-1}. We want the particles emerging from slit S_2 into region II to have a fixed velocity of 1000 ms^{-1}. What should be the value of the uniform magnetic field in region I ? (Ans. 0.1 T)
11. A proton, a deutron and an alpha particle, after being accelerated through the same potential difference, enter a region of uniform magnetic field

$\vec{B}$, in a direction perpendicular to $\vec{B}$. Compare their kinetic energies. If the radius of proton's circular path is 5 cm, what will be the radii of the paths of deutron and alpha particle ?

(Ans. $1:1:2$, $r_d = 7.07$ cm, $r_\alpha = 10$ cm)

12. A particle having a charge of 5.0 μC and a mass of 5.0×10^{-12} kg is projected with a velocity of 1.0 km s^{-1} in a magnetic field of magnitude 5.0 mT. The angle between the magnetic field and the velocity is $\sin^{-1}(0.90)$. Show that the path of the particle will be a helix. Find the diameter of the helix and its pitch. (Ans. 36 cm, 55 cm)

HINTS

1. Use $\frac{e}{m} = \frac{v}{rB}$.

2. Proceed as in Example 45 on page 4.36.

3. $T = \frac{2\pi m}{qB} = \frac{2\pi \times 10 \times 10^{-6}}{100 \times 10^{-6} \times 25 \times 10^{-3}} = \mathbf{25\ s.}$

4. Use $B = \frac{\sqrt{2\,meV}}{er}$ and $T = \frac{2\pi m}{eB}$.

5. As $eE = evB$

$\therefore \quad v = \frac{E}{B} = \frac{1.0 \times 10^4}{2 \times 10^{-3}} = \mathbf{5 \times 10^6\ ms^{-1}}$

When electric field is removed, electrons follow circular path.

$\therefore \quad \frac{mv^2}{r} = evB$

or $\quad r = \frac{mv}{eB} = \frac{9.1 \times 10^{-31} \times 5 \times 10^6}{1.6 \times 10^{-19} \times 2 \times 10^{-3}}$

$= 1.43 \times 10^{-2}\,\text{m} = \mathbf{1.43\ cm.}$

6. As $T = \frac{2\pi m}{eB} \quad \therefore B = \frac{2\pi m}{eT}$

7. Here $\frac{1}{2}mv^2 = 62.5\,\text{eV} = 62.5 \times 1.6 \times 10^{-19}$ J

$= 10^{-17}$ J

$v = \sqrt{\frac{2 \times 10^{-17}}{m}}$

$= \sqrt{\frac{2 \times 10^{-17}}{9.1 \times 10^{-31}}} = 4.69 \times 10^6\ \text{ms}^{-1}$

$B = \frac{mv}{er} = \frac{9.1 \times 10^{-31} \times 4.69 \times 10^6}{1.6 \times 10^{-19} \times 5 \times 10^{-2}}$

$= \mathbf{5.335 \times 10^{-4}\ T.}$

8. $\frac{1}{2}mv^2 = eV \quad$ or $\quad v = \sqrt{\frac{2eV}{m}}$

$\therefore \quad r = \frac{mv}{eB} = \frac{m}{eB}\sqrt{\frac{2eV}{m}} = \frac{\sqrt{2\,meV}}{eB}$

$= \frac{\sqrt{2 \times 9 \times 10^{-31} \times 1.6 \times 10^{-19} \times 2000}}{1.6 \times 10^{-19} \times 0.2}$

$= 7.5 \times 10^{-4}$ m $= \mathbf{0.75\ mm.}$

9. $B = \frac{mv}{qr} = \frac{1.67 \times 10^{-27} \times 10^7}{1.6 \times 10^{-19} \times 6.4 \times 10^6} = \mathbf{1.63 \times 10^{-8}\ T.}$

10. For the particles to go undeflected,

Force due to electric field = Force due to magnetic field

$qE = qvB$

or $\quad B = \frac{E}{v} = \frac{100\ \text{Vm}^{-1}}{1000\ \text{ms}^{-1}} = \mathbf{0.1\ T.}$

11. For a given p.d., the kinetic energy of a charged particle is proportional to its charge.

$\therefore \quad K_p : K_d : K_\alpha = e : e : 2e = \mathbf{1:1:2}$

Radius of the circular path of any particle of kinetic energy K,

$r = \frac{mv}{qB} = \frac{m}{qB}\cdot\sqrt{\frac{2K}{m}} = \frac{\sqrt{2mK}}{qB}$

$\therefore$ For proton, $r_p = \frac{\sqrt{2m_p K_p}}{eB} = 5$ cm

For deutron, $r_d = \frac{\sqrt{2m_d K_d}}{eB} = \frac{\sqrt{2 \times 2m_p \times K_p}}{eB}$

$= \sqrt{2}\,r_p = 1.414 \times 5\,\text{cm} = \mathbf{7.07\ cm.}$

For α-particle, $r_\alpha = \frac{\sqrt{2m_\alpha K_d}}{2eB} = \frac{\sqrt{2 \times 4 \times m_p \times 2K_p}}{2eB}$

$= 2r_p = \mathbf{10\ cm.}$

12. Here $q = 5.0\,\mu\text{C} = 5 \times 10^{-6}$ C, $m = 5 \times 10^{-12}$ kg,

$v = 1.0\ \text{km s}^{-1} = 10^3\ \text{ms}^{-1}$, $B = 5.0\ \text{mT} = 5 \times 10^{-3}$ T

As $\theta = \sin^{-1}(0.90)$, so $\sin\theta = 0.90$

$\cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - 0.81} = \sqrt{0.19} = 0.436$

$v_\perp = v\sin\theta = 10^3 \times 0.90 = 0.9 \times 10^3\ \text{ms}^{-1}$

$v_\parallel = v\cos\theta = 10^3 \times 0.436 = 4.36 \times 10^2\ \text{ms}^{-1}$

Velocity component $v_\parallel$ moves the electron along the field and $v_\perp$ along circular path. Hence the motion is helical.

Diameter $= 2r = \frac{2mv_\perp}{qB} = \frac{2 \times 5 \times 10^{-12} \times 0.9 \times 10^3}{5 \times 10^{-6} \times 5 \times 10^{-3}}$

$= 0.36$ m $= \mathbf{36\ cm.}$

$T = \frac{2\pi r}{v_\perp} = \frac{3.14 \times 0.36}{0.9 \times 10^3} = 1.25 \times 10^{-3}$ s

Pitch $= v_\parallel \times T = 4.36 \times 10^2 \times 1.25 \times 10^{-3}$

$= 0.55$ m $= \mathbf{55\ cm.}$

4.16 CYCLOTRON

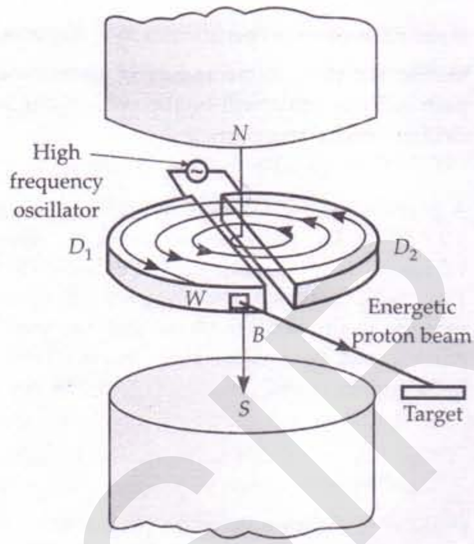
17. *What is a cyclotron ? Discuss the principle, construction, theory and working of a cyclotron. What is the maximum kinetic energy acquired by the accelerated charged particles ? Give the limitations and uses of a cyclotron.*

Cyclotron. *It is a device used to accelerate charged particles like protons, deutrons, α-particles, etc., to very high energies.* It was invented by *E.O. Lawrence* and *M.S. Livingston* in 1934 at Berkeley, California University.

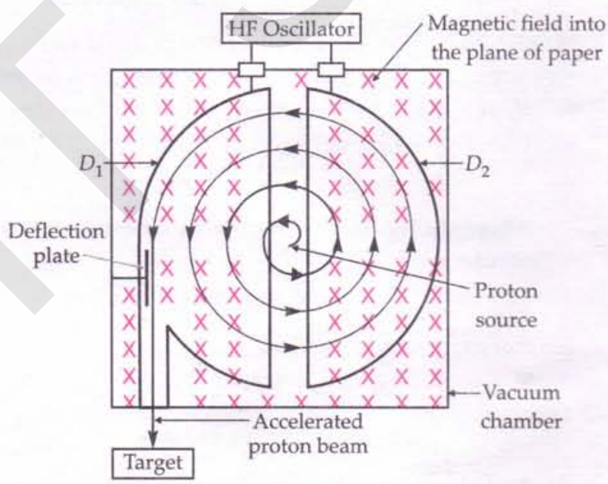
Principle. *A charged particle can be accelerated to very high energies by making it pass through a moderate electric field a number of times. This can be done with the help of a perpendicular magnetic field which throws the charged particle into a circular motion, the frequency of which does not depend on the speed of the particle and the radius of the circular orbit.*

Construction. As shown in Fig. 4.68, a cyclotron consists of the following main parts :

1. It consists of two small, hollow, metallic half-cylinders D_1 and D_2, called *dees* as they are in the shape of D.
2. They are mounted inside a vacuum chamber between the poles of a powerful electromagnet.
3. The dees are connected to the source of high frequency alternating voltage of few hundred kilovolts.
4. The beam of charged particles to be accelerated is injected into the dees near their centre, in a plane perpendicular to the magnetic field.
5. The charged particles are pulled out of the dees by a deflecting plate (which is negatively charged) through a window *W*.
6. The whole device is in high vacuum (pressure ~ 10^{-6} mm of Hg) so that the air molecules may not collide with the charged particles.



(a)



(b)

Fig. 4.68 Cyclotron (*a*) Front view (*b*) Section diagram.

Theory. Let a particle of charge q and mass m enter a region of magnetic field $\vec{B}$ with a velocity $\vec{v}$, normal to the field $\vec{B}$. The particle follows a circular path, the necessary centripetal force being provided by the magnetic field. Therefore,

Magnetic force on charge q

= Centripetal force on charge q

or $q v B \sin 90° = \frac{mv^2}{r}$ or $r = \frac{mv}{qB}$

Period of revolution of the charged particle is given by

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v} \cdot \frac{mv}{qB} = \frac{2\pi m}{qB}$$

Hence frequency of revolution of the particle will be

$$f_c = \frac{1}{T} = \frac{qB}{2\pi m}$$

Clearly, this frequency is *independent* of both the velocity of the particle and the radius of the orbit and is called ***cyclotron frequency*** or ***magnetic resonance frequency***. This is the key fact which is made use of in the operation of a cyclotron.

Working. Suppose a positive ion, say a proton, enters the gap between the two dees and finds dee D_1 to be negative. It gets accelerated towards dee D_1. As it enters the dee D_1, it does not experience any electric field due to shielding effect of the metallic dee. The perpendicular magnetic field throws it into a circular path. At the instant the proton comes out of dee D_1, it finds dee D_1 positive and dee D_2 negative. It now gets accelerated towards dee D_2. It moves faster through D_2 describing a larger semicircle than before. Thus if the frequency of the applied voltage is kept exactly the same as the frequency of revolution of the proton, then every time the proton reaches the gap between the two dees, the electric field is reversed and proton receives a push and finally it acquires very high energy. This is called the cyclotron's *resonance condition*. The proton follows a spiral path. The accelerated proton is ejected through a window by a deflecting voltage and hits the target.

Maximum K.E. of the accelerated ions. The ions will attain maximum velocity near the periphery of the dees. If v_0 is the maximum velocity acquired by the ions and r_0 is the radius of the dees, then

$$\frac{mv_0^2}{r_0} = qv_0 B \quad \text{or} \quad v_0 = \frac{qBr_0}{m}$$

The maximum kinetic energy of the ions will be

$$K_0 = \frac{1}{2}mv_0^2 = \frac{1}{2}m\left(\frac{qBr_0}{m}\right)^2$$

or

$$K_0 = \frac{q^2B^2r_0^2}{2m}$$

Limitations of cyclotron :

1. According to the Einstein's *special theory of relativity*, the mass of a particle increases with the increase in its velocity as

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

where m_0 is the rest mass of the particle. At high velocities, the cyclotron frequency ($f_c = qB/2\pi m$) will decrease due to increase in mass. This will throw the particles out of resonance with the oscillating field. That is, as the ions reach the gap between the dees, the polarity of the dees is not reversed at that instant. Consequently the ions are not accelerated further.

The above drawback is overcome either by increasing magnetic field as in a ***synchrotron*** or by decreasing the frequency of the alternating electric field as in a ***synchro-cyclotron***.

2. *Electrons cannot be accelerated in a cyclotron.* A large increase in their energy increases their velocity to a very large extent. This throws the electrons out of step with the oscillating field.

3. *Neutrons*, being electrically neutral, cannot be accelerated in a cyclotron.

Uses of cyclotron :

1. The high energy particles produced in a cyclotron are used to bombard nuclei and study the resulting nuclear reactions and hence investigate nuclear structure.
2. The high energy particles are used to produce other high energy particles, such as neutrons, by collisions. These fast neutrons are used in atomic reactors.
3. It is used to implant ions into solids and modify their properties or even synthesise new materials.
4. It is used to produce radioactive isotopes which are used in hospitals for diagnosis and treatment.

For Your Knowledge

- As the magnetic force on a charged particle acts perpendicular to the velocity, it does not do any work on the particle. As a result, the kinetic energy or the speed of the particle does not change due to the magnetic force.
- When a charged particle is projected into a uniform magnetic field with its initial velocity perpendicular to the field, the magnetic force acts on the charged particle perpendicular to both the magnetic field and its direction of motion. This force produces centripetal force to make the particle move in a circle in a plane perpendicular to the magnetic field.
- When a charged particle moves perpendicular to a uniform magnetic field : (*i*) its path is circular in a plane perpendicular to the magnetic field and its direction of motion, (*ii*) the radius of the circular path is proportional to its momentum, (*iii*) the kinetic energy and speed of the particle do not change, (*iv*) the force acting on the particle is independent of the radius of the circular orbit but is proportional to its speed *i.e.*, $F \propto r^\circ$ and $F \propto v$ and (*v*) the period of revolution of the charged particle is independent of its speed and the radius of its circular orbit.
- When a charged particle is projected into a uniform magnetic field at an arbitrary angle with the field, the component of the initial velocity parallel to the magnetic field will make the particle move along the direction of the field while the perpendicular component will compel it to follow a circular path. As a result, the particle will follow a helical path with its axis parallel to the field.
- In a cyclotron, it is the electric field which accelerates the charged particles. The magnetic field does not change the speed, it only makes the charged particle to cross the same electric field again and again by making it move along a circular path.

Examples based on Cyclotron

Formulae Used

For the accelerated charged particle,

1. Velocity, $v = \frac{qBr}{m}$
2. Period of revolution, $T = \frac{2\pi m}{qB}$
3. Cyclotron frequency, $f_c = \frac{qB}{2\pi m}$
4. Maximum kinetic energy, $K_{max} = \frac{q^2 B^2 R^2}{2\,m}$

   where R is the radius of the dees.

Units Used

B is in tesla, v in ms^{-1}, r in metre, T in second and f_c in Hz.

Example 51. *Deutrons are accelerated in a cyclotron that has an oscillatory frequency of 10^7 Hz and a dee radius of 50 cm. (i) What is the strength of the magnetic field needed to accelerate the deutrons ? (ii) What is the energy of deutrons emerging from the cyclotron. Mass of a deutron $= 3.34 \times 10^{-27}$ kg and charge of a deutron $= 1.6 \times 10^{-19}$ C.*

Solution. $\nu = 10^7$ Hz, $R = 50$ cm $= 0.50$ m, $m = 3.34 \times 10^{-27}$ kg, $q = 1.6 \times 10^{-19}$ C

(*i*) Cyclotron frequency, $f_c = \frac{qB}{2\pi m}$

$$\therefore \quad B = \frac{2\pi m f_c}{q} = \frac{2 \times 3.14 \times 3.34 \times 10^{-27} \times 10^7}{1.6 \times 10^{-19}}$$

$$= \mathbf{1.3\ T.}$$

(*ii*)
$$K_{max} = \frac{q^2 B^2 R^2}{2m}$$

$$= \frac{(1.6 \times 10^{-19})^2 \times (1.3)^2 \times (0.50)^2}{2 \times 3.34 \times 10^{-27}}$$

$$= 1.62 \times 10^{-12}\ \text{J} = \frac{1.62 \times 10^{-12}}{1.6 \times 10^{-13}}\ \text{MeV}$$

$$= \mathbf{10.125\ MeV.}$$

Example 52. *A cyclotron's oscillator frequency is 10 MHz. What should be the operating magnetic field for accelerating protons ? If the radius of the 'dees' is 60 cm, what is the kinetic energy of the proton beam produced by the accelerator ? ($e = 1.60 \times 10^{-19}$ C, $m_p = 1.67 \times 10^{-27}$ kg). Express your answer in units of MeV (1 MeV $= 1.602 \times 10^{-13}$ J).*

[CBSE OD 05 ; NCERT]

Solution. Here $f_c = 10$ MHz $= 10^7$ Hz,

$e = 1.6 \times 10^{-19}$ C, $R = 60$ cm $= 0.6$ m,

$m_p = 1.67 \times 10^{-27}$ kg

The operating magnetic field for accelerating protons is

$$B = \frac{2\pi m_p f_c}{e} = \frac{2 \times 3.14 \times 1.67 \times 10^{-27} \times 10^7}{1.6 \times 10^{-19}}$$

$$= \mathbf{0.66\ T.}$$

Kinetic energy of the emerging beam will be

$$K_{max} = \frac{e^2 B^2 R^2}{2\,m_p} = \frac{(1.6 \times 10^{-19})^2 \times (0.66)^2 \times (0.6)^2}{2 \times 1.67 \times 10^{-27}}$$

$$= 1.2 \times 10^{-12}\ \text{J} = \frac{1.2 \times 10^{-12}}{1.602 \times 10^{-13}}\ \text{MeV}$$

$$= \mathbf{7.4\ MeV.}$$

Example 53. *In a cyclotron, a magnetic induction of 1.4 T is used to accelerate protons. How rapidly should the electric field between the dees be reversed ? The mass and charge of proton are 1.67×10^{-27} kg and 1.6×10^{-19} C respectively.*

Solution. Here $B = 1.4$ T, $m = 1.67 \times 10^{-27}$ kg,

$e = 1.6 \times 10^{-19}$ C

The time required by a charged particle to complete a semicircle in a dee is

$$t = \frac{\pi m}{eB} = \frac{3.14 \times 1.67 \times 10^{-27}}{1.6 \times 10^{-19} \times 1.4} = \mathbf{2.34 \times 10^{-8}\ s}$$

Thus the direction of electric field should reverse after every 2.34×10^{-8} s.

The frequency of the applied electric field should be

$$f_c = \frac{1}{2t} = \frac{1}{2 \times 2.34 \times 10^{-8}} = \mathbf{2.14 \times 10^7\ Hz.}$$

Example 54. *If the maximum value of accelerating potential provided by a radio frequency oscillator be 20 kV, find the number of revolutions made by a proton in a cyclotron to achieve one fifth of the speed of light. Mass of a proton $= 1.67 \times 10^{-27}$ kg.*

Solution. In a cyclotron, a proton gains energy eV when it crosses a region of potential difference V. In one revolution, the particle crosses the gap twice. So the energy gained in each revolution $= 2\,eV$.

Suppose the particle makes n revolutions before emerging from the dees. The gain in its kinetic energy will be

$$\frac{1}{2} mv^2 = 2\,neV \quad \text{or} \quad n = \frac{mv^2}{4\,eV}$$

Given $v = \frac{c}{5} = \frac{3 \times 10^8}{5} = 0.6 \times 10^8\ ms^{-1}$,

$m = 1.67 \times 10^{-27}$ kg

$$\therefore \quad n = \frac{1.67 \times 10^{-27} \times (0.6 \times 10^8)^2}{4 \times 1.6 \times 10^{-19} \times 20 \times 10^3} = \mathbf{470\ revolutions.}$$

Problems For Practice

1. An electron of energy 10,000 eV describes a circular path in a plane at right angles to a uniform magnetic field of 0.01 T. (*a*) What is the radius of the circular orbit ? (*b*) What is the cyclotron frequency ? (*c*) What is the period of its revolution ? (*d*) What is the direction of revolution as viewed by an observer looking in the direction of the field ?
(Ans. 3.4×10^{-2} m, 2.8×10^{8} s^{-1}, 3.6×10^{-9} s, clockwise sense).
2. The protons are accelerated by a cyclotron, when a magnetic field of 2.0 T is applied perpendicular to the plane of the dees. Calculate the energy of the proton in MeV, if the circular path of the protons has a radius of 40 cm before the protons leave the cyclotron. Given mass of a proton $= 1.67\times10^{-27}$ kg.
(Ans. 30.6 MeV)
3. A cyclotron has an oscillatory frequency of 12 MHz and a dee radius of 50 cm. Calculate the magnetic field required to accelerate deutrons of mass 3.3×10^{-27} kg and charge 1.6×10^{-19} C. What is the energy of the deutrons emerging from the cyclotron ?
(Ans. 1.56 T, 14.7 MeV)
4. Alpha particles of mass 6.68×10^{-27} kg and charge 3.2×10^{-19} C are accelerated in a cyclotron in which a magnetic field of 1.25 T is applied perpendicular to the dees. How rapidly should the electric field between the dees be reversed ? What are the velocity and kinetic energy of an alpha particle when it moves in a circular orbit of radius 25 cm ?
(**Ans.** 9.5×10^{6} Hz, 1.5×10^{7} ms^{-1}, 7.5×10^{-13} J)

HINTS

2. $$K_{max} = \frac{q^2B^2R^2}{2m} = \frac{(1.6\times10^{-19})^2\times(2.0)^2\times(0.40)^2}{2\times1.67\times10^{-27}}\ \text{J}$$
$$= \frac{(1.6\times10^{-19})^2\times(2.0)^2\times(0.40)^2}{2\times1.67\times10^{-27}\times1.6\times10^{-13}}\ \text{MeV}$$
$$= \mathbf{30.6\ MeV.}$$

3. As $f_c = \dfrac{qB}{2\pi m}$
$$\therefore\quad B = \frac{2\pi m f_c}{q} = \frac{2\times3.142\times3.3\times10^{-27}\times12\times10^{6}}{1.6\times10^{-19}}$$
$$= \mathbf{1.56\ T.}$$
$$K_{max} = \frac{q^2B^2R^2}{2m} = \frac{(1.6\times10^{-19})^2\times(1.56)^2\times(0.50)^2}{2\times3.3\times10^{-27}}\ \text{J}$$
$$= \frac{(1.6\times10^{-19})^2\times(1.56)^2\times(0.50)^2}{2\times3.3\times10^{-27}\times1.6\times10^{-13}}\ \text{MeV}$$
$$= \mathbf{14.7\ MeV.}$$

4. $$t = \frac{\pi m}{qB} = \frac{3.14\times6.68\times10^{-27}}{3.2\times10^{-19}\times1.25} = 5.25\times10^{-8}\ \text{s}$$
Direction of electric field should be reversed after every 5.25×10^{-8} s.
Applied frequency,
$$f_c = \frac{1}{2t} = \frac{1}{2\times5.25\times10^{-8}} = \mathbf{9.5\times10^{6}\ Hz}$$
As $$r = \frac{mv}{qB}$$
$$\therefore\quad v = \frac{rqB}{m} = \frac{0.25\times3.2\times10^{-19}\times1.25}{6.68\times10^{-27}}$$
$$= \mathbf{1.5\times10^{7}\ ms^{-1}}$$
$$K = \frac{1}{2}mv^2$$
$$= \frac{1}{2}\times6.68\times10^{-27}\times(1.5\times10^{7})^2$$
$$= \mathbf{7.5\times10^{-13}\ J.}$$

4.17 FORCE ON A CURRENT CARRYING CONDUCTOR IN A MAGNETIC FIELD

18. *Describe an experiment to illustrate that a current carrying conductor experiences a force in a magnetic field. What is the cause of this force ?*

Force on a current carrying conductor in a magnetic field. When a conductor carrying a current is placed in an external magnetic field, it experiences a mechanical force. To demonstrate this force, take a small aluminium rod AB. Suspend it horizontally by means of connecting wires from a stand, as shown in Fig. 4.69.

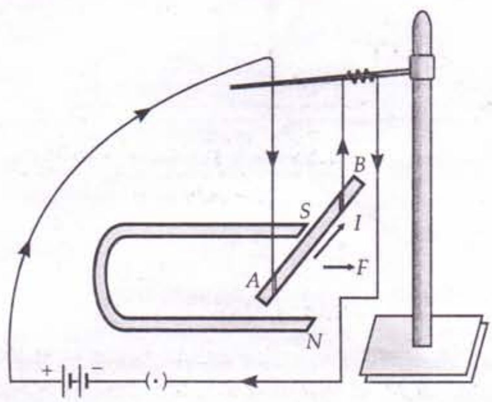


Fig. 4.69 Force on a current in a magnetic field.

Place a strong horse-shoe magnet in such a way that the rod is between the two poles with the field directed upwards. Now, if a current is passed through the rod from A to B, the rod gets deflected to the right. If we reverse the direction of current or interchange the poles of the magnet, the deflection of the rod is also reversed. The direction of force is perpendicular to both the current and the magnetic field and is given by *Fleming's left hand rule.*

Cause of the force on a current carrying conductor in a magnetic field. A current is an assembly of moving charges and a magnetic field exerts a force on a moving charge. That is why a current carrying conductor when placed in a magnetic field experiences a sideways force as the force experienced by the moving charges (free electrons) is transmitted to the conductor as a whole.

19. *Derive an expression for the force experienced by a current carrying straight conductor placed in a magnetic field. Under what conditions, is this force*

(i) zero and (ii) maximum ?

State the rule used to determine the direction of this force ?

Expression for the force on a current carrying conductor in a magnetic field. As shown in Fig. 4.70, consider a conductor PQ of length l, area of cross-section A, carrying current I along $+ve$ Y-direction. The field $\vec{B}$ acts along $+ve$ Z-direction. The electrons drift towards left with velocity $\vec{v}_d$. Each electron experiences a magnetic Lorentz force along $+ve$ X-axis, which is given by

$$\vec{f} = -e(\vec{v}_d \times \vec{B})$$

If n is the number of free electrons per unit volume, then total number of electrons in the conductor is

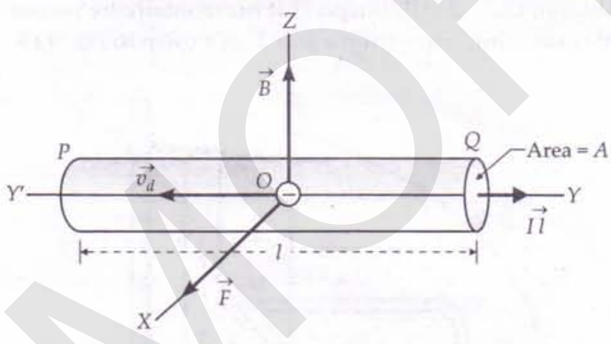


Fig. 4.70 Force on a current in a magnetic field.

$$N = n \times \text{volume} = nAl$$

Total force on the conductor is

$$\vec{F} = N\vec{f} = nAl[-e(\vec{v}_d \times \vec{B})]$$

$$= enA[-l\vec{v}_d \times \vec{B}]$$

If $I\vec{l}$ represents a current element vector in the direction of current, then vectors $\vec{l}$ and $\vec{v}_d$ will have opposite directions and we can take

$$-l\vec{v}_d = v_d\vec{l}$$

$$\therefore \quad \vec{F} = enAv_d(\vec{l} \times \vec{B})$$

But $enAv_d$ = current, I

Hence $\vec{F} = I(\vec{l} \times \vec{B})$

Magnitude of force. The magnitude of the force on the current carrying conductor is given by

$$F = IlB\sin\theta$$

where θ is the angle between the direction of the magnetic field and the direction of flow of current.

Special Cases

(*i*) If $\theta = 0°$ or $180°$, then

$$F = IlB(0) = 0$$

Thus a current carrying conductor placed parallel to the direction of the magnetic field does not experience any force.

(*ii*) If $\theta = 90°$, then

$$F = IlB\sin 90° = IlB$$

or $F_{max} = IlB$

Thus a *current carrying conductor placed perpendicular to the direction of a magnetic field experiences a **maximum force**.*

Direction of force. The direction of force on a current carrying conductor placed in a perpendicular magnetic field is given by ***Fleming's left hand rule.*** *Stretch the thumb and the first two fingers of the left hand in mutually perpendicular directions. If the forefinger points in the direction of the magnetic field, central finger in the direction of current, then the thumb gives the direction of force on the conductor.* In Fig. 4.70, the field $\vec{B}$ is along + Z-direction, the current I along + Y-direction and so the force $\vec{F}$ acts along + X-direction.

Examples based on
Force on a Current Carrying Conductor in a Magnetic Field

Formulae Used

1. $\vec{F} = I(\vec{l} \times \vec{B})$ **2.** $F = IlB\sin\theta$

3. $F_{max} = IlB$

Units Used

Force F is in newton, current I in ampere, length l in metre and field B in tesla.

Example 55. *A wire of length l carries a current I along the X-axis. A magnetic field $\vec{B} = B_0(\hat{i} + \hat{j} + \hat{k})$ tesla exists in space. Find the magnitude of the magnetic force on the wire.*

Solution. As the wire carries current I along the X-axis, so $\vec{l} = l\,\hat{i}$

Also, $\vec{B} = B_0(\hat{i} + \hat{j} + \hat{k})$ tesla

Magnetic force on the wire is

$$\vec{F} = I(\vec{l} \times \vec{B}) = I[l\,\hat{i} \times B_0(\hat{i} + \hat{j} + \hat{k})]$$
$$= B_0 Il[\hat{i} \times (\hat{i} + \hat{j} + \hat{k})]$$
$$= B_0 Il[\hat{i} \times \hat{i} + \hat{i} \times \hat{j} + \hat{i} \times \hat{k}]$$
$$= B_0 Il(\vec{0} + \hat{k} - \hat{j}) = (\hat{k} - \hat{j}) B_0 Il$$

Magnitude of the magnetic force is

$$F = \sqrt{1^2 + (-1)^2}\, B_0 Il = \sqrt{2}\, \mathbf{B_0 I l} \textbf{ newton.}$$

Example 56. *The horizontal component of the earth's magnetic field at a certain place is 3.0×10^{-5} T and the direction of the field is from the geographic south to the geographic north. A very long straight conductor is carrying a steady current of 1 A. What is the force per unit length on it when it is placed on a horizontal table and the direction of the current is (a) east to west, (b) south to north ?* **[NCERT]**

Solution. The force on a conductor of length l placed in a magnetic field B, and carrying current I, is

$$F = IlB \sin\theta$$

The force per unit length will be

$$f = \frac{F}{l} = IB \sin\theta$$

where θ is the angle that the conductor makes with the direction of $\vec{B}$.

(*a*) When the current flows east to west, $\theta = 90°$.

$$\therefore \quad f = IB \sin 90° = 1 \times 3.0 \times 10^{-5} \times 1$$
$$= \mathbf{3.0 \times 10^{-5}\ Nm^{-1}}$$

According to Fleming's left hand rule, this force acts *vertically downwards.*

(*b*) When the current flows from south to north, $\theta = 0°$

$$\therefore \quad f = Il \sin 0° = \mathbf{0}$$

Thus the force per unit length of the conductor is zero.

Example 57. *A current of 2 A enters at the corner 'a' of a square frame of side 20 cm and leaves at opposite corner 'c'. A magnetic field of B = 0.25 T acts in a direction perpendicular to the plane of paper, as shown in Fig. 4.71. Find the magnitude and direction of the magnetic forces on the four sides of the frame.*

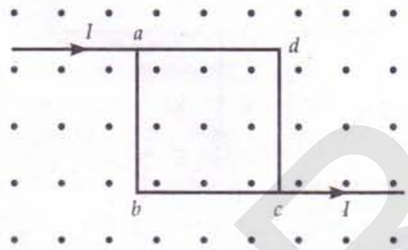


Fig. 4.71

Solution. By symmetry, the current through each of the four sides will be 1 A. Also,

$$l = 20 \text{ cm} = 0.20 \text{ m}, \; B = 0.25 \text{ T}$$

Magnitude of force on each side is

$$F = IlB \sin 90°$$
$$= 1 \times 0.20 \times 0.25 \times 1 = 0.05 \text{ N}$$

By Fleming's left hand rule, forces on *ab* and *dc* will be towards left and on *ad* and *bc* downward.

Example 58. *A magnetic field of 1.0 T is produced by an electromagnet in a cylindrical region of radius 4.0 cm, as shown in Fig. 4.72. A wire, carrying current of 2.0 A, is placed perpendicular to and intersecting the axis of the cylindrical region. Find the magnetic force acting on the wire.*

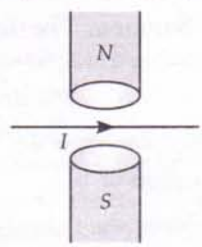


Fig. 4.72

Solution. Clearly, the magnetic field acts vertically downwards while the current flows horizontally, so

$$\theta = 90°.$$

Length of the wire in the cylindrical region

$$= 2r = 2 \times 4.0 \text{ cm} = 0.08 \text{ m}$$

$$\therefore \quad F = IlB \sin 90° = 2.0 \times 0.08 \times 1.0 \times 1 = \mathbf{0.16\ N}$$

This force acts on the wire normally into the plane of paper.

Example 59. *A straight wire of mass 200 g and length 1.5 m carries a current of 2 A. It is suspended in mid-air by a uniform horizontal magnetic field $\vec{B}$. What is the magnitude of the magnetic field ?* **[NCERT ; CBSE F 15]**

Solution. Suppose that a wire AB carries a current of 2 A in the direction as shown in Fig. 4.73. The weight mg of the wire acts vertically downwards. Therefore, according to Fleming's left hand rule, the magnetic field $\vec{B}$ must act perpendicularly into the plane of paper so that the magnetic force $\vec{F}$ on the wire acts vertically upwards.

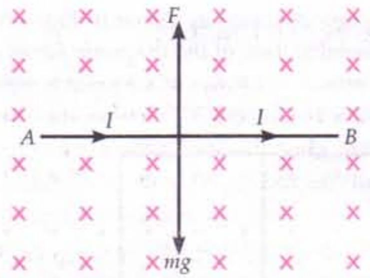


Fig. 4.73

For mid-air suspension,

Magnetic force on the wire = Weight of the wire

$$IlB \sin 90° = mg$$

or $$B = \frac{mg}{Il}$$

But $m = 200 \text{ g} = 0.2 \text{ kg}$, $g = 9.8 \text{ ms}^{-2}$, $l = 1.5 \text{ m}$, $I = 2 \text{ A}$

$$\therefore \quad B = \frac{0.2 \times 9.8}{1.5 \times 2} = \mathbf{0.65 \text{ T}}.$$

Example 60. *What is the force on a wire of length 4.0 cm placed inside a solenoid near its centre, making an angle of 60° with its axis ? The wire carries a current of 12 A and the magnetic field due to the solenoid has a magnitude of 0.25 T.*

Solution. The force on a conductor of length l placed in a magnetic field B, and carrying current I, is

$$F = IlB \sin \theta$$

where θ is the angle that the conductor makes with the direction of $\vec{B}$.

Since the field due to a solenoid near its centre is along its axis, so $\theta = 60°$.

Also $I = 12$ A, $l = 4.0 \text{ cm} = 0.04 \text{ m}$, $B = 0.25$ T

$$\therefore \quad F = 12 \times 0.04 \times 0.25 \sin 60° = \mathbf{0.10 \text{ N}}.$$

Example 61. *On a smooth plane inclined at 30° with the horizontal, a thin current-carrying metallic rod is placed parallel to the horizontal ground. The plane is located in a uniform magnetic field of 0.15 T in the vertical direction. For what value of current can the rod remain stationary ? The mass per unit length of the rod is 0.03 kg m^{-1}.* [NCERT]

Solution. Suppose a rod PQ is placed horizontally on an inclined plane as shown in Fig. 4.74. Various forces acting on the current carrying rod PQ are

(*i*) its weight Mg acting vertically downwards ; and

(*ii*) horizontal force BIl due to the magnetic field $\vec{B}$.

In order that the rod remains stationary, the component of the weight of the rod along the inclined plane must be balanced by the component of the force BIl along the inclined plane, *i.e.*,

$$Mg \sin \theta = BIl \cos \theta$$

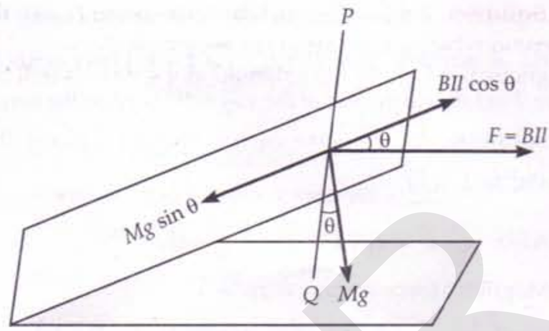


Fig. 4.74

If m is the mass per unit length of the rod, then

$$M = ml$$

$$\therefore \quad mlg \sin \theta = BIl \cos \theta$$

or $$I = \frac{mg \tan \theta}{B} = \frac{0.30 \times 9.8 \times \tan 30°}{0.15}$$

$$= \frac{0.30 \times 9.8 \times 0.5774}{0.15} = \mathbf{11.32 \text{ A}}.$$

Example 62. *A short conductor of length 5.0 cm is placed parallel to a long conductor of length 1.5 m near its centre. The conductors carry currents 4.0 A and 3.0 A respectively in the same direction. What is the total force experienced by the long conductor when they are 3.0 cm apart ?* [NCERT]

Solution. As the two conductors have different lengths, the longer conductor may be considered to be of infinite length. Therefore, magnetic field produced by it at a distance of 3 cm (0.03 m) is given by

$$B = \frac{\mu_0 I_2}{2\pi r} = \frac{4\pi \times 10^{-7} \times 3}{2\pi \times 0.03} \text{ T} = 2 \times 10^{-5} \text{ T}$$

Force on the short conductor due to this magnetic field will be

$$F = I_1 l_1 B = 4 \times 5 \times 10^{-2} \times 2 \times 10^{-5} \text{ N}$$

$$= \mathbf{4 \times 10^{-6} \text{ N}}$$

According to Newton's third law, the longer conductor will also experience a force of reaction equal to 4.0×10^{-6} N. As the currents are in the same direction, the force is attractive.

Example 63. *A solenoid 60 cm long and of radius 4.0 cm has 3 layers of windings of 300 turns each. A 2.0 cm long wire of mass 2.5 g lies inside the solenoid near its centre normal to its axis ; both the wire and the axis of the solenoid are in the horizontal plane. The wire is connected through two leads parallel to the axis of the solenoid to an external battery which supplies a current of 6.0 A in the wire. What value of current in the windings of the solenoid can support the weight of the wire ? $g = 9.8 \text{ ms}^{-2}$.* [NCERT]

Solution. Let I be the current in the windings of the solenoid which can support the weight of the wire. The magnetic field inside the solenoid along its axis will be

$$B = \mu_0 n I$$

Now $$n = \frac{\text{Total number of turns}}{\text{Length of the solenoid}}$$

$$= \frac{300 \times 3}{60 \times 10^{-2}} = 1500 \text{ turns m}^{-1}$$

$$\therefore \quad B = 4\pi \times 10^{-7} \times 1500 \times I = 6\pi \times 10^{-4} I \text{ tesla}$$

This field acts perpendicular to the current carrying wire, therefore, the magnetic force on the wire will be

$$F = I' l B = 6 \times (2 \times 10^{-2}) \times 6\pi \times 10^{-4}\ I \text{ newton}$$

The current I would support the wire if the above force equals the weight of the wire,

i.e., $$6 \times 2 \times 10^{-2} \times 6\pi \times 10^{-4} I = 2.5 \times 10^{-3} \times 9.8$$

or $$I = \frac{2.5 \times 10^{-3} \times 9.8}{72 \times 3.14 \times 10^{-6}} \text{ A} = \mathbf{108.3\ A}.$$

Example 64. *Figure 4.75 shows a triangular loop PQR carrying current I. The triangle is equilateral with side equal to l. If a uniform magnetic field B exists parallel to PQ, then find the forces acting on the three wires separately.*

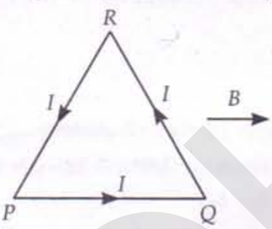


Fig. 4.75

Solution. As $\vec{B} \parallel PQ$, so force on wire PQ is

$$\vec{F}_1 = I\ \vec{PQ} \times \vec{B}$$

or $$F_1 = I \times PQ \times B \times \sin 0° = 0$$

Force on wire QR,

$$\vec{F}_2 = I\ \vec{QR} \times \vec{B}$$

or $$F_2 = IlB \sin 120° = \frac{\sqrt{3}}{2}\ IlB$$

By right hand rule, this force acts normally into the plane of paper.

Force on wire RP,

$$\vec{F}_3 = I\ \vec{RP} \times \vec{B}$$

or $$F_3 = IlB \sin 120° = \frac{\sqrt{3}}{2}\ IlB$$

This force acts normally out of the plane of paper.

Problems For Practice

1. A current of 1 A flows in a wire of length 0.1 m in a magnetic field of 0.5 T. Calculate the force acting on the wire when the wire makes an angle of (*a*) 90° (*b*) 0°, with respect to the magnetic field. (Ans. 0.05 N, 0)
2. A current of 5.0 A is flowing upward in a long vertical wire placed in a uniform horizontal northward magnetic field of 0.02 T. How much force and in what direction will the field exert on 0.06 m length of the wire ? (Ans. 6×10^{-3} N, towards west)
3. What is the magnitude of force on a wire of length 0.04 m placed inside a solenoid near its centre, making an angle of 30° with its axis ? The wire carries a current of 12 A and the magnetic field due to the solenoid is of magnitude 0.25 T. **[CBSE OD 90 C]** (Ans. 0.06 N)
4. A long straight conductor P carrying a current of 2 A is placed parallel to a short conductor Q of length 0.05 m carrying a current of 3 A. The two conductors are 0.10 m apart.

   Calculate

   (*a*) the magnetic field due to P at Q

   (*b*) the approximate force on Q.

   (Ans. 4×10^{-6} T, 6×10^{-7} N)
5. A straight wire 1 m long carries a current of 10 A at right angles to a uniform magnetic field of 1 Wb m^{-2}. Find the mechanical force on the wire and the power required to move it at 15 ms^{-1} in a plane at right angles to the field. (Ans. 10 N, 150 W)
6. A wire AB making an angle of 30° with a horizontal is supported by a magnetic field of 0.65 T, directed normally into the plane of paper. If the wire carries a current of 5 A, determine its mass per unit length. (Ans. 0.2872 kg m^{-1})

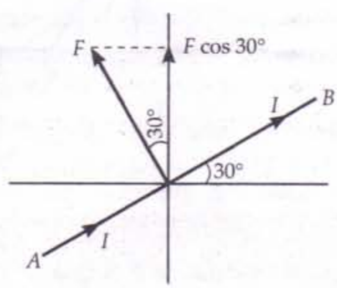


Fig. 4.76

7. A horizontal wire 0.1 m long carries a current of 5 A. Find the magnitude and direction of the magnetic field which can support the weight of wire assuming that its mass is 3×10^{-3} kg m^{-1}.
(Ans. 5.88×10^{-3} T)

8. A conductor of length 10 cm is placed perpendicular to a uniform magnetic field of strength 100 oersted. If a charge of 5 C passes through it in 5 s, find the force experienced by the conductor.
(Ans. 10^{-3} N)

9. A conductor of length 20 cm is placed (*i*) parallel (*ii*) perpendicular (*iii*) inclined at an angle 30°, to a uniform magnetic field of 2 T. If a charge of 10 C passes through it in 5 s, find the force experienced by the conductor. [Ans. (*i*) zero (*ii*) 0.8 N (*iii*) 0.4 N]

10. A current of 5.0 A exists in the loop shown in Fig. 4.77. The wire *AB* has a length of 50 cm and lies in a magnetic field of 0.20 T. What is the magnetic force acting on the wire ?
(Ans. 0.50 N, towards the inside of the loop)

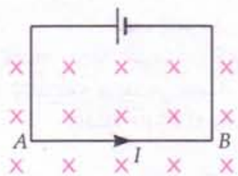


Fig. 4.77

11. A horizontal wire 0.1 m long having mass 3 g carries a current of 5 A. Find the magnitude of the magnetic field which must act at 30° to the length of the wire in order to support its weight ?
(Ans. 0.1176 T)

12. Find the magnitude of the magnetic force on the segment *PQ* placed in a magnetic field of 0.25 T, if a current of 5 A flows through it, as shown in Fig. 4.78. (Ans. 0.32 N)

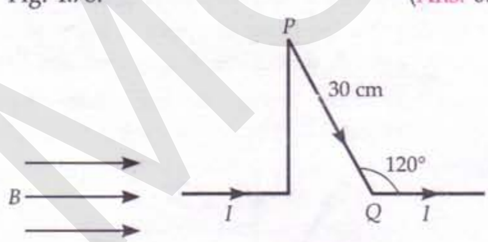


Fig. 4.78

HINTS

3. $F = IlB \sin \theta = 12 \times 0.04 \times 0.25 \sin 30° = \mathbf{0.06\ N}$.

4. (*a*) Magnetic field due to *P* at *Q* is
$$B = \frac{\mu_0 I}{2\pi a} = \frac{4\pi \times 10^{-7} \times 2}{2\pi \times 0.10} = \mathbf{4 \times 10^{-6}\ T.}$$

(*b*) Force on *Q*, $F = IlB \sin \theta$
$$= 3 \times 0.05 \times 4 \times 10^{-6} \times \sin 90°$$
$$= \mathbf{6.0 \times 10^{-7}\ T.}$$

5. $F = IlB \sin 90° = 10 \times 1 \times 1 \times 1 = \mathbf{10\ N}$
$P = Fv = 10 \times 15 = \mathbf{150\ W.}$

6. Force on wire *AB*, $F = IlB \sin 90° = IlB$
Component of the force in the vertically upward direction $= F \cos 30° = IlB \cdot \frac{\sqrt{3}}{2}$
If *m* is the mass per unit length of wire, then its weight $= mlg$
$$\therefore \quad mlg = IlB \cdot \frac{\sqrt{3}}{2}$$
$$\text{or} \quad m = \frac{IB \cdot \sqrt{3}}{2g} = \frac{5 \times 0.65 \times \sqrt{3}}{2 \times 9.8}$$
$$= \mathbf{0.2872\ kg\ m^{-1}}.$$

7. In equilibrium,
Magnetic force on wire = Weight of wire
$$\text{or} \quad IlB \sin 90° = mg$$
$$\text{or} \quad B = \frac{m}{l} \cdot \frac{g}{I} = \frac{3 \times 10^{-3} \times 9.8}{5} = \mathbf{5.88 \times 10^{-3}\ T.}$$

8. $F = IlB \sin \theta = \frac{q}{t} \cdot lB \sin 90°$
$$= \frac{5 \times 0.10 \times 100 \times 10^{-4} \times 1}{5} = \mathbf{10^{-3}\ N.}$$

9. Proceed as in Problem 8 above.

10. $F = IlB \sin \theta = 5.0 \times 0.50 \times 0.20 \times \sin 90° = \mathbf{0.50\ N.}$

11. $F = IlB \sin \theta = mg$
$$\therefore \quad B = \frac{mg}{Il \sin \theta} = \frac{3 \times 10^{-3} \times 9.8}{0.1 \times 5 \times \sin 30°} = \mathbf{0.1176\ T.}$$

12. $F = IlB \sin \theta = 5 \times 0.30 \times 0.25 \sin (180° - 120°)$
$= 5 \times 0.30 \times 0.25 \times \sin 60° = \mathbf{0.32\ N.}$

4.18 FORCES BETWEEN TWO PARALLEL CURRENT-CARRYING CONDUCTORS

20. *How will you show experimentally the existence of (i) attractive forces between parallel currents and (ii) repulsive forces between anti-parallel currents ?*

Forces between two parallel current-carrying conductors. It was first observed by *Ampere* in 1820 that two parallel straight conductors carrying currents in the same direction attract each other and those carrying currents in the opposite directions repel each other.

Experiment 1. As shown in Fig. 4.79, the upper ends of two wires are connected to the –ve terminal of a battery and their lower ends are connected to the +ve terminal of the battery through a mercury bath. When the circuit is completed, the current flows in the two wires in the same direction. The two wires are found to be closer to each other, indicating a force of attraction between them.

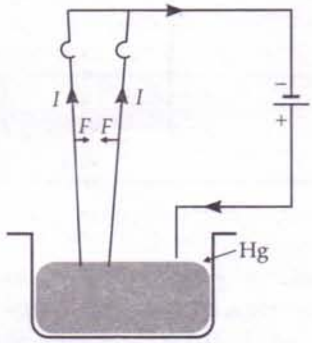


Fig. 4.79 Attractive force between parallel currents.

Experiment 2. As shown in Fig. 4.80, two wires are connected to a battery through a mercury bath in such a way that current flows in them in succession. When the circuit is closed, the currents in the two wires flow in opposite directions. The two wires move away from each other, indicating a force of repulsion between them.

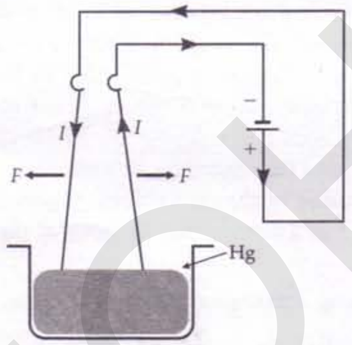


Fig. 4.80 Repulsive force between antiparallel currents.

21. *Derive an expression for the force per unit length between two infinitely long straight parallel current carrying wires. Hence define one ampere. Also define coulomb in terms of ampere.*

Expression for the force between two parallel current-carrying wires. As shown in Fig. 4.81(*a*), consider two long parallel wires *AB* and *CD* carrying currents I_1 and I_2. Let r be the separation between them.

The magnetic field produced by current I_1 at any point on wire *CD* is

$$B_1 = \frac{\mu_0 I_1}{2\pi r}$$

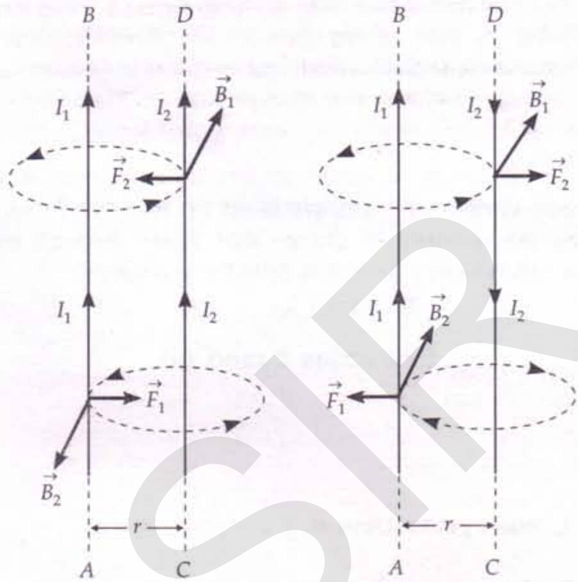


Fig. 4.81 (*a*) Parallel currents attract, (*b*) Antiparallel currents repel.

This field acts perpendicular to the wire *CD* and points into the plane of paper. It exerts a force on current carrying wire *CD*. The force acting on length l of the wire *CD* will be

$$F_2 = I_2 l B_1 \sin 90° = I_2 l \cdot \frac{\mu_0 I_1}{2\pi r} = \frac{\mu_0 I_1 I_2}{2\pi r} \cdot l$$

Force per unit length,

$$f = \frac{F_2}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

According to Fleming's left hand rule, this force acts at right angles to *CD*, towards *AB* in the plane of the paper. Similarly, an equal force is exerted on the wire *AB* by the field of wire *CD*. Thus when the currents in the two wires are in the same direction, the forces between them are attractive. It can be easily seen that

$$\vec{F}_1 = -\vec{F}_2$$

As shown in Fig. 4.81(*b*), when the currents in the two parallel wires flow in opposite directions (anti-parallel), the forces between the two wires are repulsive. Thus,

Parallel currents attract and antiparallel currents repel.

Definition of ampere.

When $I_1 = I_2 = 1$ A and $r = 1$ m, we get

$$f = \frac{\mu_0}{2\pi} = 2 \times 10^{-7} \text{ Nm}^{-1}$$

One ampere is that value of steady current, which on flowing in each of the two parallel infinitely long conductors of negligible cross-section placed in vacuum at a distance of 1 m from each other, produces between them a force of 2×10^{-7} newton per metre of their length.

Definition of coulomb in terms of ampere. If a steady current of 1 ampere is set up in a conductor, then the quantity of charge that flows through its cross-section in 1 second is called one coulomb.

$$1\,C = 1\,As$$

Examples based on Forces between Parallel Current-Carrying Wires

Formulae Used

1. Force per unit length, $f = \frac{\mu_0 I_1 I_2}{2\pi r}$

2. Force on length l of one of the wires,

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi r}$$

Units Used

Force is in newton, currents in ampere, distance r in metre and field B in tesla.

Constant Used

$\mu_0 = 4\pi \times 10^{-7}\ \text{TmA}^{-1}$.

Example 65. *A current of 5.0 A flows through each of two parallel long wires. The wires are 2.5 cm apart. Calculate the force acting per unit length of each wire. Use the standard value of constant required. What will be the nature of the force, if both currents flow in the same direction ?* [Punjab 99]

Solution. Here $I_1 = I_2 = 5$ A,

$r = 2.5$ cm $= 2.5 \times 10^{-2}$ m, $\mu_0 = 4\pi \times 10^{-7}\ \text{TmA}^{-1}$

Force acting per unit length of each wire,

$$f = \frac{\mu_0 I_1 I_2}{2\pi r} = \frac{4\pi \times 10^{-7} \times 5 \times 5}{2\pi \times 2.5 \times 10^{-2}}$$

$$= \mathbf{2 \times 10^{-4}\ Nm^{-1}}$$

As the currents in both the wires flow in the same direction, the force will be ***attractive***.

Example 66. *A long horizontal wire P carries a current of 50 A. It is rigidly fixed. Another fine wire Q is placed directly above and parallel to P. The weight of the wire Q is 0.075 Nm^{-1} and it carries a current of 25 A. Find the position of the wire Q from the wire P so that Q remains suspended due to the magnetic repulsion. Also indicate the direction of current in Q with respect to P.* [Roorkee 96]

Solution. The magnetic force per unit length on the wire Q due to the current in wire P is

$$F = \frac{\mu_0}{2\pi} \cdot \frac{I_1 I_2}{r}$$

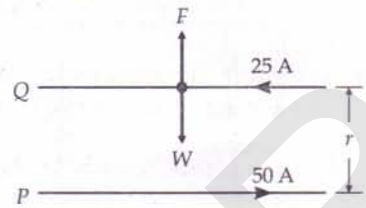


Fig. 4.82

The currents in P and Q must have opposite directions, only then Q will experience a repulsive force which would balance the weight of Q.

$$\therefore \quad F = \frac{\mu_0 I_1 I_2}{2\pi r} = W$$

or
$$r = \frac{\mu_0}{2\pi} \cdot \frac{I_1 I_2}{W} = \frac{2 \times 10^{-7} \times 50 \times 25}{0.075}$$

$$= 3.33 \times 10^{-3}\ \text{m} = \mathbf{3.33\ mm.}$$

Example 67. *A current balance (or ampere balance) is a device for measuring currents. The current to be measured is arranged to go through two long parallel wires of equal length in opposite directions one of which is linked to the pivot of the balance. The resulting repulsive force on the wire is balanced by putting a suitable mass in the scale pan hanging on the other side of the pivot. In one measurement, the mass in the scale pan is 30.0 g, the length of the wires is 50.0 cm each, and the separation between them is 10.0 mm. What is the value of the current being measured ? Take $g = 9.80\ ms^{-2}$ and assume that the arms of the balance are equal.* [NCERT]

Solution. $m = 30.0$ g $= 0.03$ kg, $l = 50$ cm $= 0.50$ m, $r = 10.0$ mm $= 0.01$ m, $g = 9.8\ \text{ms}^{-2}$

Force per unit length between two parallel conductors,

$$f = \frac{\mu_0}{2\pi} \cdot \frac{I_1 I_2}{r}$$

$\therefore$ Force on a conductor of length l,

$$F = \frac{\mu_0}{2\pi} \cdot \frac{I_1 I_2 l}{r}$$

When the pan is balanced,

Weight in scale pan = Balancing force

i.e.,
$$mg = \frac{\mu_0}{2\pi} \cdot \frac{I \times I}{r} \cdot l$$

or
$$I^2 = \frac{2\pi\, mgr}{\mu_0 l} = \frac{2\pi \times 0.03 \times 9.8 \times 0.01}{4\pi \times 10^{-7} \times 0.50}$$

$$= 29400$$

$$\therefore \quad I = \sqrt{29400} = \mathbf{171.46\ A.}$$

Example 68. *A rectangular loop of sides 25 cm and 10 cm carrying a current of 15 A is placed with its longer side parallel to a long straight conductor 2.0 cm apart carrying a current of 25 A. What is the net force on the loop ?* [CBSE OD 05]

Solution. Consider the rectangular loop *ABCD* placed near a long straight conductor *XY*, as shown in Fig. 4.83. The arm *AB* will get attracted, while *CD* will get repelled. Forces on arms *BC* and *AD*, being equal, opposite and collinear, will cancel each other.

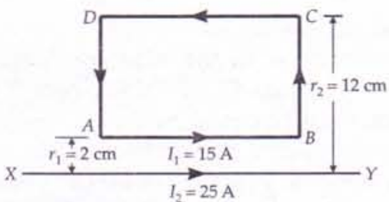


Fig. 4.83

Current through the rectangular loop, $I_1 = 15$ A

Current through the long wire *XY*, $I_2 = 25$ A

Force on *AB*,

$$F_1 = \frac{\mu_0}{4\pi} \cdot \frac{2 I_1 I_2}{r_1} \times \text{length of conductor } AB$$

$$= \frac{10^{-7} \times 2 \times 15 \times 25}{2.0 \times 10^{-2}} \times 25 \times 10^{-2}$$

$$= 9.375 \times 10^{-4} \text{N} \qquad (\textit{Attractive})$$

Force on *CD*,

$$F_2 = \frac{\mu_0}{4\pi} \cdot \frac{2 I_1 I_2}{r_2} \times \text{length of conductor } CD$$

$$= \frac{10^{-7} \times 2 \times 15 \times 25}{12.0 \times 10^{-2}} \times 25 \times 10^{-2}$$

$$= 1.5625 \times 10^{-4} \qquad (\textit{Repulsive})$$

∴ Net force on the loop,

$$F = F_1 - F_2 = 9.375 \times 10^{-4} - 1.5625 \times 10^{-4}$$

$$= 7.8125 \times 10^{-4} \text{ N} \simeq \mathbf{7.8 \times 10^{-4} \text{ N}} \text{ (Attractive)}$$

Thus the force on the loop will act towards the long conductor (attractive) if the current in its closer side is in the same direction as the current in the long conductor, otherwise it will be repulsive.

Example 69. *In Fig. 4.84, the wires AB, CD and EF are long and have identical resistances. The separation between the neighbouring wires is 1.0 cm. The wires AE and BF have negligible resistance and the ammeter reads 15 A. Calculate the magnetic force per unit length of AB and CD.*

Solution. By symmetry, current through each of the wires *AB*, *CD* and *EF* is 5 A.

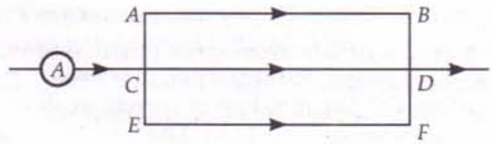


Fig. 4.84

Force per unit length of *AB* due to current in *CD* is

$$f_1 = \frac{\mu_0 I_1 I_2}{2\pi r} = \frac{4\pi \times 10^{-7} \times 5 \times 5}{2\pi \times 1.0 \times 10^{-2}}$$

$$= 5.0 \times 10^{-4} \text{ Nm}^{-1}, \text{ directed downward}$$

Force per unit length of *AB* due to current in *EF* is

$$f_2 = \frac{4\pi \times 10^{-7} \times 5 \times 5}{2\pi \times 2.0 \times 10^{-2}}$$

$$= 2.5 \times 10^{-4} \text{ Nm}^{-1}, \text{ directed downward}$$

Total force per unit length of *AB* is

$$f = f_1 + f_2$$

$$= \mathbf{7.5 \times 10^{-4} \text{ Nm}^{-1}}, \text{ directed downward}$$

Force per unit length of *CD* due to current in *AB* is

$$f_1 = \frac{4\pi \times 10^{-7} \times 5 \times 5}{2\pi \times 1.0 \times 10^{-2}}$$

$$= 5.0 \times 10^{-4} \text{ Nm}^{-1}, \text{ directed upward}$$

Force per unit length of *CD* due to current in *EF* is

$$f_2 = \frac{4\pi \times 10^{-7} \times 5 \times 5}{2\pi \times 1.0 \times 10^{-2}}$$

$$= 5.0 \times 10^{-4} \text{ Nm}^{-1}, \text{ directed downward}$$

∴ Total force per unit length of *CD* = **zero.**

Problems For Practice

1. A long horizontal rigidly supported wire carries a current of 100 A. Directly above it and parallel to it is a fine wire that carries a current of 200 A and weighs 0.05 Nm^{-1}. How far above the wire should the second wire be kept to support it by magnetic repulsion ? (Ans. 8 cm)
2. A wire *AB* is carrying a steady current of 12 A and is lying on the table. Another wire *CD* carrying 5 A is held directly above *AB* at a height of 1 mm. Find the mass per unit length of the wire *CD* so that it remains suspended at its position when left free. Give the direction of the current flowing in *CD* with respect to that in *AB*.
[Take the value of $g = 10 \text{ ms}^{-2}$]. **[CBSE OD 13]**
(Ans. 1.2×10^{-3} kg m^{-1}, in the opposite direction)
3. Two very long, straight, parallel wires *A* and *B* carry currents of 10 A and 20 A respectively, and

are at a distance 20 cm apart, as shown in Fig. 4.85. If a third wire *C* (length 15 cm) having a current of 10 A is placed between them, how much force will act on *C* ? The direction of current in all the three wires is same. (Ans. 3.0×10^{-5} N, towards *B*)

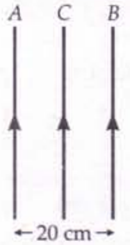


Fig. 4.85

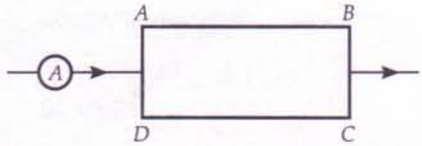


Fig. 4.86

4. In Fig. 4.86, *ABCD* is a rectangular loop made of uniform wire. The length $AD = BC = 1\,cm$. The sides *AB* and *DC* are much longer than *AD* or *BC*. Find the magnetic force per unit length acting on the wire *DC* due to the wire *AB* if the ammeter reads 10 A. (Ans. 5×10^{-4} Nm^{-1}, attractive)

5. A rectangular loop of wire of size 2cm × 5 cm carries a steady current of 1 A. A straight long wire carrying 4 A current is kept near the loop as shown in Fig. 4.87. If the loop and the wire are coplanar, find (*i*) the torque acting on the loop and (*ii*) the magnitude and direction of the force on the loop due to the current carrying wire. **[CBSE D 12]**

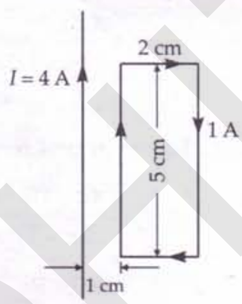


Fig. 4.87

[Ans. (*i*) $\tau = 0$ (*ii*) $F = 2.67\ \mu N$, towards the straight wire]

6. A square loop of side 20 cm carrying current of 1 A is kept near an infinite long straight wire carrying a current of 2 A in the same plane as shown in Fig. 4.88. Calculate the magnitude and direction of the net force exerted on the loop due to the current carrying conductor. **[CBSE OD 15C]**

(Ans. 5.33×10^{-7} N)

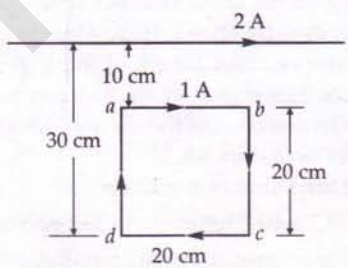


Fig. 4.88

HINTS

1. Force of repulsion per unit length,

$$f = \frac{4\pi \times 10^{-7} \times 100 \times 200}{2\pi \times r} = 0.05\ \text{Nm}^{-1}$$

$$\therefore \quad r = \frac{4 \times 10^{-3}}{0.05}\ \text{m} = \mathbf{8\ cm.}$$

2. Weight per unit length of upper wire

= Magnetic force per unit length

$$\frac{W}{l} = \frac{F}{l} \quad \Rightarrow \quad \frac{mg}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

Mass per unit length

$$= \frac{m}{l} = \frac{\mu_0 I_1 I_2}{2\pi r g} = \frac{4\pi \times 10^{-7} \times 12 \times 5}{2\pi \times 10^{-3} \times 10}$$

$$= \mathbf{1.2 \times 10^{-3}\ kgm^{-1}}$$

The direction of current in *CD* must be opposite to that of current in *AB* so that the force between the two wires is repulsive.

3. Force on *C* due to *A*,

$$F_1 = \frac{4\pi \times 10^{-7} \times 10 \times 10 \times 0.15}{2\pi \times 0.10}$$

$= 3.0 \times 10^{-5}$ N, towards *A*

Force on *C* due to *B*,

$$F_2 = \frac{4\pi \times 10^{-7} \times 20 \times 10 \times 0.15}{2\pi \times 0.10}$$

$= 6.0 \times 10^{-5}$ N, towards *B*

Net force on *C*

$= F_2 - F_1 = \mathbf{3.0 \times 10^{-5}\ N}$, towards *B*.

4. $$f = \frac{\mu_0 I_1 I_2}{2\pi r} = \frac{4\pi \times 10^{-7} \times 5 \times 5}{2\pi \times 0.01}$$

$= \mathbf{5 \times 10^{-4}\ Nm^{-1}}$, attractive

5. (*i*) As direction of the magnetic field due to the straight conductor is parallel to the area vector (both normal to the plane of the loop), so torque $\tau = 0$.

(*ii*) Proceed as in Example 68 on page 4.51

6. $$F = \frac{\mu_0 I_1 I_2}{2\pi r} l$$

$\therefore$ Net force on sides *ab* and *cd*

$$= \frac{\mu_0 I_1 I_2}{2\pi} l \left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$= \frac{4\pi \times 10^{-7} \times 2 \times 1 \times 20 \times 10^{-2}}{2\pi}\left[\frac{1}{10 \times 10^{-2}} - \frac{1}{30 \times 10^{-2}}\right]\text{N}$$

$$= 4 \times 10^{-7} \times 20\left[\frac{20}{10 \times 30}\right]\text{N} = \mathbf{5.33 \times 10^{-7}\ N}$$

This force is directed towards the infinitely long straight wire.

4.19 TORQUE EXPERIENCED BY A CURRENT LOOP IN A UNIFORM MAGNETIC FIELD

22. *Derive an expression for the torque acting on a current carrying loop suspended in a uniform magnetic field.*

Torque on a current loop in a uniform magnetic field. As shown in Fig. 4.89(a), consider a rectangular coil $PQRS$ suspended in a uniform magnetic field $\vec{B}$, with its axis perpendicular to the field.

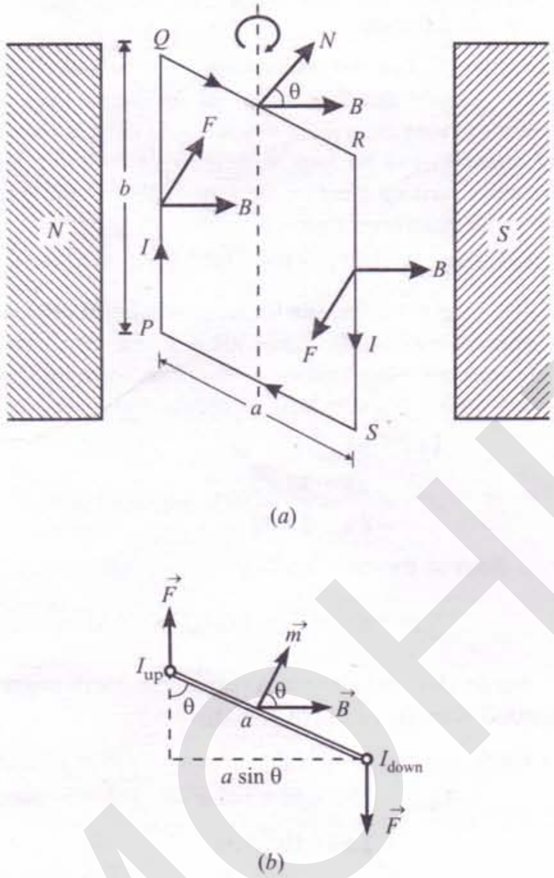


Fig. 4.89 (a) A rectangular loop $PQRS$ in a uniform magnetic field $\vec{B}$. (b) Top view of the loop, magnetic dipole moment $\vec{m}$ is shown.

Let I = current flowing through the coil $PQRS$

a, b = sides of the coil $PQRS$

$A = ab$ = area of the coil

θ = angle between the direction of $\vec{B}$ and normal to the plane of the coil.

According to Fleming's left hand rule, the magnetic forces on sides PS and QR are equal, opposite and collinear (along the axis of the loop), so their resultant is zero.

The side PQ experiences a normal inward force equal to IbB while the side RS experiences an equal normal outward force. These two forces form a couple which exerts a torque given by

$$\tau = \text{Force} \times \text{perpendicular distance}$$

$$= IbB \times a \sin\theta = IBA \sin\theta$$

If the rectangular loop has N turns, the torque increases N times *i.e.*,

$$\tau = NIBA \sin\theta$$

But $NIA = m$, the magnetic moment of the loop, so

$$\tau = mB \sin\theta$$

In vector notation, the torque $\vec{\tau}$ is given by

$$\vec{\tau} = \vec{m} \times \vec{B}$$

The direction of the torque $\vec{\tau}$ is such that it rotates the loop clockwise about the axis of suspension.

Special Cases

(i) When $\theta = 0°, \tau = 0$, *i.e.*, the torque is *minimum* when the plane of the loop is perpendicular to the magnetic field.

(ii) When $\theta = 90°, \tau = NIBA$, *i.e.*, the torque is *maximum* when the plane of the loop is parallel to the magnetic field. Thus

$$\tau_{max} = NIBA$$

For Your Knowledge

- The expression for torque ($\tau = NIBA \sin\theta$) holds for a planar loop of any shape. Thus the *torque* on a planar current loop depends on current, strength of magnetic field and area of the loop. It is *independent of the shape of the loop.*
- For a planar current loop of a given perimeter suspended in a magnetic field, the torque is maximum when the loop is circular in shape. This is because for a given perimeter, a circle has maximum area.
- The expression $\vec{\tau} = \vec{m} \times \vec{B}$ for the torque on a current loop in a magnetic field is analogous to the expression $\vec{\tau} = \vec{p}_e \times \vec{E}$ for the torque on an electric dipole in an electric field. This supports the fact that a *current loop is a magnetic dipole.*

- The torque on a current loop in a magnetic field is the operating principle of the electric motor and most electric meters used for measuring currents and voltages, called galvanometers.
- If the direction of the magnetic field makes an angle α with the plane of the current loop, then

$$\theta + \alpha = 90° \text{ or } \theta = 90° - \alpha$$

$$\therefore \quad \tau = NIBA \sin(90° - \alpha) = NIBA \cos \alpha.$$

- In a uniform magnetic field, the net magnetic force on a current loop is zero but torque acting on it may be zero or non-zero.
- In a non-uniform magnetic field, the net magnetic force on a current is non-zero but torque acting on it may be zero or non-zero.

Examples based on Torques on Current Loops

Formulae Used

Torque on a current loop in a magnetic field,

$$\tau = NIBA \sin \theta = mB \sin \theta$$

where $m = NIA$ = magnetic dipole moment of the current loop.

In vector form, $\vec{\tau} = \vec{m} \times \vec{B}$.

Units Used

Current I is in ampere, area A in m^2, field B in tesla, torque τ in Nm and magnetic moment m in Am^2.

Example 70. *The maximum torque acting on a coil of effective area 0.04 m^2 is 4×10^{-8} Nm when the current in it is 100 μA. Find the magnetic induction in which it is kept.*

Solution. $A = 0.04\ m^2$, $\tau_{max} = 4 \times 10^{-8}$ Nm,

$$I = 100\ \mu A = 10^{-4}\ A, \quad N = 1$$

As $\quad \tau_{max} = NIBA$

$\therefore$ Magnetic induction,

$$B = \frac{\tau_{max}}{NIA} = \frac{4 \times 10^{-8}}{1 \times 10^{-4} \times 0.04} = \mathbf{10^{-2}\ Wb\ m^{-2}}.$$

Example 71. *Calculate the torque on a 100 turn rectangular coil of length 40 cm and breadth 20 cm, carrying a current of 10 A, when placed making an angle of 60° with a magnetic field of 3 T.*

Solution. Here $I = 10$ A, $N = 100$, $B = 3$ T,

$$A = 40 \text{ cm} \times 20 \text{ cm} = 800 \text{ cm}^2 = 8 \times 10^{-2}\ m^2$$

$$\theta = 90° - 60° = 30°$$

= Angle between $\vec{B}$ and the normal to the plane of the coil

$\therefore$ Torque,

$$\tau = NIBA \sin \theta = 100 \times 10 \times 3 \times 8 \times 10^{-2} \times \sin 30° = \mathbf{120\ Nm}.$$

Example 72. *Given a uniform magnetic field of 100 G in an east to west direction and a 44 cm long wire with a current carrying capacity of almost 10 A. What is the shape and orientation of the loop made of this wire which yields maximum turning effect on the loop ? What is the magnitude of the maximum torque ?*

Solution. $B = 100\ G = 100 \times 10^{-4}$ T, $\quad I = 10$ A

The torque on the planar loop will be maximum if its area is maximum. Since for a given perimeter, a circle encloses maximum area, therefore, the wire should be bent into a circular loop of radius r given by

$$2\pi r = 44$$

$$r = \frac{44}{2\pi} = \frac{44 \times 7}{2 \times 22} = 7 \text{ cm} = 0.07 \text{ m}$$

$\therefore$ Area of the circular loop,

$$A = \pi r^2 = \frac{22}{7} \times (0.07)^2 = 0.0154\ m^2$$

Again, for maximum torque, the loop must be oriented with its plane in N-S direction.

Then

$$\tau_{max} = IBA = 10 \times 100 \times 10^{-4} \times 0.0154 \text{ Nm} = \mathbf{1.54 \times 10^{-3}\ Nm}.$$

Example 73. *A circular coil of 25 turns and radius 6.0 cm, carrying a current of 10 A, is suspended vertically in a uniform magnetic field of magnitude 1.2 T. The field lines run horizontally in the plane of the coil. Calculate the force and the torque on the coil due to the magnetic field. In which direction should a balancing torque be applied to prevent the coil from turning ?*

Solution. Consider any element $\vec{dl}$ of the wire. Force on this element is $I\,\vec{dl} \times \vec{B}$. For each element $\vec{dl}$, there is another element $-\vec{dl}$ on the current loop. Forces on each pair of such elements cancel out. Hence net force on the coil in a uniform magnetic field is zero.

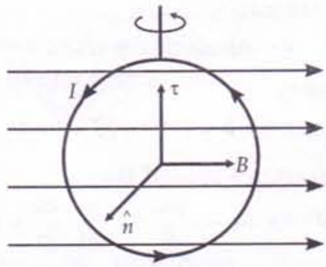


Fig. 4.90

In Fig. 4.90, $\hat{n}$ is a unit vector normal to the plane of the loop, directed outward. The angle between $\hat{n}$ and $\vec{B}$ is 90°. The magnitude of the torque acting on the loop is

$$\tau = NIBA \sin\theta$$
$$= 25 \times 10 \times 1.2 \times \pi (0.06)^2 \times \sin 90°$$
$$= \mathbf{3.4\ Nm}$$

This torque acts in the vertically upward direction producing turning effect in the direction of curved arrow. To prevent the coil from turning, a balancing torque $\tau' = \tau$ must be applied.

Example 74. *A rectangular coil of sides 8 cm and 6 cm having 2000 turns and carrying a current of 200 mA is placed in a uniform magnetic field of 0.2 T directed along the + ve X-axis.*

(i) *What is the maximum torque the coil can experience ? In which orientation does it experience the maximum torque ?*

(ii) *For which orientations of the coil is the torque zero ? When is this equilibrium stable and when is it unstable ?* **[NCERT]**

Solution. $l = 8$ cm $= 0.08$ m, $b = 6$ cm $= 0.06$ m

$N = 2000$, $I = 200$ mA $= 0.2$ A, $B = 0.2$ T

The magnitude of the magnetic dipole moment is given by

$$m = NIA = 2000 \times 0.2 \times (0.08 \times 0.06) = 1.92\ \text{Am}^2$$

The direction of $\vec{m}$ is normal to area $\vec{A}$ of the coil from *S*-pole to *N*-pole. Magnitude of torque on the coil is

$$\tau = mB \sin\theta$$

For maximum torque, $\vec{m}$ must be perpendicular to $\vec{B}$.

Therefore,

$$\tau_{max} = mB = 1.92 \times 0.2 = \mathbf{0.384\ Nm}$$

Thus the torque on the coil is maximum whenever the *X*-axis lies in the plane of the coil.

The torque on the coil is zero when $\vec{m}$ is parallel or antiparallel to $\vec{B}$, *i.e.*, when it lies in the *YZ*-plane. The coil will be in stable equilibrium when $\vec{m}$ is parallel to $\vec{B}$ and in unstable equilibrium when $\vec{m}$ is antiparallel to $\vec{B}$.

Example 75. *A 100-turns coil kept in a magnetic field $\vec{B} = 0.05\ Wb\ m^{-2}$, carries a current of 1 A, as shown in Fig. 4.91. Find the torque acting on the coil.* **[MNREC 97]**

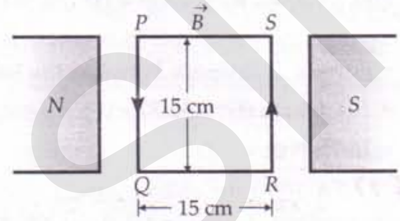


Fig. 4.91

Solution. Here the angle between the axis of rotation of the coil and the magnetic field $\vec{B}$ is 90°.

$\therefore$ $N = 100$, $I = 1$ A,

$A = 15\ \text{cm} \times 15\ \text{cm} = 225 \times 10^{-4}\ \text{m}^2$,

$B = 0.5\ \text{Wb m}^{-2}$, $\theta = 90°$

Torque,

$$\tau = NIBA \sin\theta$$
$$= 100 \times 1 \times 0.5 \times 225 \times 10^{-4} \times \sin 90°$$
$$= \mathbf{1.125\ Nm}$$

As the force on the arm *PQ* acts upwards and that on *SR* downwards, so the torque acts anticlockwise.

Example 76. *A parallelogram-shaped coil PQRS of sides 0.7 m and 0.5 m carries a current of 1.5 A, as shown in Fig. 4.92. It is placed in a magnetic field $\vec{B} = 40$ T parallel to PS. Find (i) forces on the sides of the coil and (ii) torque on the coil.*

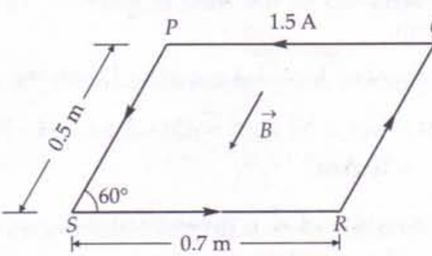


Fig. 4.92

Solution. As the magnetic field $\vec{B}$ acts parallel to sides PS and QR, no forces act on these sides.

Force on side PQ is

$$F = IlB \sin\theta = 1.5 \times 0.7 \times 40 \times \sin 60^\circ$$
$$= 1.5 \times 0.7 \times 40 \times 0.866 = \mathbf{36.37\ N}$$

According to Fleming's left hand rule, the force F will act normally upward.

Similarly, force on side SR will also be 36.37 N, but it will be directed normally inward.

(*ii*) As the forces on the sides PQ and SR are equal, opposite and parallel, they form a couple which exerts a torque.

$$\tau = \text{Force} \times \perp \text{ distance between the two forces}$$
$$= F \times PS \sin 60^\circ = 36.37 \times 0.5 \times 0.866$$
$$= \mathbf{15.75\ Nm.}$$

Example 77. *A 100 turn closely wound circular coil of radius 10 cm carries a current of 3.2 A. (i) What is the field at the center of the coil ? (ii) What is the magnetic moment of this arrangement ?*

The coil is placed in a vertical plane and is free to rotate about a horizontal axis which coincides with its diameter. A uniform magnetic field of 2 T in the horizontal direction exists such that initially the axis of the coil is in the direction of the field. The coil rotates through an angle of 90° under the influence of the magnetic field. (iii) What are the magnitudes of the torques on the coil in the initial and final position ? (iv) What is the angular speed acquired by the coil when it has rotated by 90° ? The M.I. of the coil is 0.1 kg m². **[NCERT]**

Solution. (*i*) Here $N = 100$, $I = 3.2$ A,

$$r = 10 \text{ cm} = 0.1 \text{ m}$$

Magnetic field at the centre of the coil,

$$B = \frac{\mu_0 NI}{2r} = \frac{4\pi \times 10^{-7} \times 100 \times 3.2}{2 \times 0.1} \quad [\because \pi \times 3.2 \simeq 10]$$
$$= \frac{4 \times 10^{-5} \times 10}{2 \times 0.1} = \mathbf{2 \times 10^{-3}\ T}$$

The direction of the field is given by right hand thumb rule.

(*ii*) Magnetic moment associated with the coil,

$$m = NIA = NI \times \pi r^2 = 100 \times 3.2 \times 3.14 \times (0.1)^2$$
$$= \mathbf{10\ Am^2}$$

The direction of $\vec{m}$ is given by right hand rule.

(*iii*) Torque, $\tau = mB \sin\theta$

Initially, $\theta = 0^\circ$

$\therefore$ Initial torque,

$$\tau = mB \sin 0^\circ = \mathbf{0}$$

Final torque,

$$\tau = mB \sin 90^\circ = 10 \times 2 \times 1 = \mathbf{20\ Nm.}$$

(*iv*) By Newton's second law,

$$\tau = I\alpha = I\frac{d\omega}{dt} = I\frac{d\omega}{d\theta} \cdot \frac{d\theta}{dt} = I \cdot \frac{d\omega}{d\theta} \cdot \omega$$

But $\quad \tau = mB \sin\theta$

$$\therefore \quad I \cdot \frac{d\omega}{d\theta} \cdot \omega = mB \sin\theta \quad \text{or} \quad I\omega\, d\omega = mB \sin\theta\, d\theta$$

When θ changes from 0 to $\pi/2$, suppose the angular speed changes from 0 to ω. Integrating above equation within these limits of θ and ω, we get

$$I\int_0^{\omega} \omega\, d\omega = mB\int_0^{\pi/2} \sin\theta\, d\theta$$
$$I\left[\frac{\omega^2}{2}\right]_0^{\omega} = mB\left[-\cos\theta\right]_0^{\pi/2}$$
$$\frac{1}{2} I\omega^2 = -mB\left[\cos\frac{\pi}{2} - \cos 0\right] = mB$$

or $$\omega = \sqrt{\frac{2\,mB}{I}} = \sqrt{\frac{2 \times 10 \times 2}{0.1}} = \mathbf{20\ rad\ s^{-1}}.$$

Example 78. *A solenoid of length 0.4 m and having 500 turns of wire carries a current of 3 A. A thin coil having 10 turns of wire and of radius 0.01 m carries a current of 0.4 A. Calculate the torque required to hold the coil in the middle of the solenoid with its axis perpendicular to the axis of the solenoid* ($\mu_0 = 4\pi \times 10^{-7}$ *V-s / A-m*). **[Roorkee 90]**

Solution. For solenoid, $l = 0.4$ m, $N_1 = 500$, $I_1 = 3$ A

For coil, $N_2 = 10$, $r = 0.01$ m, $I_2 = 0.4$ A

Field inside the solenoid,

$$B = \frac{\mu_0 N_1 I_1}{l}, \text{ along the axis of solenoid.}$$

Magnetic moment of coil,

$$m = N_2 I_2 A = N_2 I_2 \pi r^2, \text{ along the axis of coil.}$$

As the axis of the coil is perpendicular to the axis of solenoid, $\vec{m}$ and $\vec{B}$ will be perpendicular to each other.

Required torque,

$$\tau = m B \sin\theta$$
$$= N_2 I_2 \pi r^2 \cdot \frac{\mu_0 N_1 I_1}{l} \cdot \sin 90^\circ$$
$$= 10 \times 0.4 \times 3\pi \times (0.01)^2 \times \frac{4\pi \times 10^{-7} \times 500 \times 3}{0.4} \times 1$$
$$= 6\pi^2 \times 10^{-7} = 6 \times 9.87 \times 10^{-7} = \mathbf{5.92 \times 10^{-6}\ Nm.}$$

Problems For Practice

1. What is the maximum torque on a rectangular coil of area 5 cm × 12 cm of 600 turns, when carrying a current of 10^{-5} A in a magnetic field of 0.10 T ? (Ans. 3.6×10^{-6} Nm)

2. What torque acts on a 40 turn coil of 100 cm^2 area carrying a current of 10 A held with its axis at right angles to a uniform magnetic of 0.2 T ? (Ans. 0.8 Nm)

3. A square shaped plane coil of area 100 cm^2 of 200 turns carries a steady current of 5 A. It is placed in a uniform magnetic field of 0.2 T acting perpendicular to the plane of the coil. Calculate the torque on the coil when its plane makes an angle of 60° with the direction of the field. In which orientation will the coil be in stable equilibrium ? [CBSE OD 15C] (Ans. 1 Nm)

4. A rectangular coil *PQRS* is placed in a uniform magnetic field $\vec{B}$, as shown in Fig. 4.93. Find the torque on the coil when it carries a current of 2.0 A. The magnitude of the field $\vec{B}$ is 2.0×10^{-2} T. (Ans. 4.0×10^{-3} Nm)

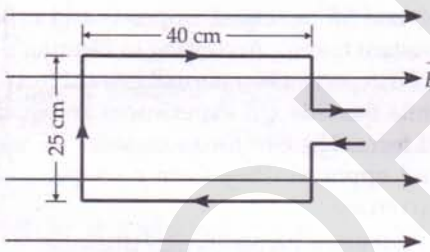


Fig. 4.93

5. A rectangular coil of 100 turns has length 5 cm and width 4 cm. It is placed with its plane parallel to a uniform magnetic field and a current of 2 A is passed through the coil. If the torque acting on the coil is 0.2 Nm, find the magnitude of the magnetic field. (**Ans.** 0.5 T)

6. A circular coil of radius 2.0 cm has 500 turns and carries a current of 1.0 A. Its axis makes an angle of 30° with the uniform magnetic field of 0.40 T that exists in the space. Find the torque acting on the coil. (**Ans.** 0.13 Nm)

7. A circular coil of 200 turns and radius 10 cm is placed in a uniform magnetic field of 0.5 T, normal to the plane of the coil. If the current in the coil is 3.0 A, calculate the (*a*) total torque on the coil. (*b*) total force on the coil. (*c*) average force on each electron in the coil, due to the magnetic field. Assume the area of cross-section of the wire to be 10^{-5} m^2 and the free electron density is $10^{29}/m^3$. [CBSE OD 08] [Ans. (*a*) zero (*b*) zero (*c*) 1.5×10^{-24} N]

HINTS

1. $A = 5\text{ cm} \times 12\text{ cm} = 60 \times 10^{-4} m^2$
$$\tau_{max} = NIBA = 600 \times 10^{-5} \times 0.10 \times 60 \times 10^{-4} = \mathbf{3.6 \times 10^{-6} Nm.}$$

2. $\tau = NIBA \sin\theta = 40 \times 10 \times 0.2 \times 100 \times 10^{-4} \sin 90° = \mathbf{0.8\ Nm}.$

3. $\tau = NIBA \sin\theta = 200 \times 5 \times 0.2 \times 100 \times 10^{-4} \sin(90° - 60°) = \mathbf{1\ Nm}.$

4. $\tau = IBA \sin\theta = 2.0 \times 2.0 \times 10^{-2} \times 0.40 \times 0.25 \times \sin 90° = \mathbf{4.0 \times 10^{-3}\ Nm}.$

5. $$B = \frac{\tau}{NIA \sin\theta} = \frac{0.2}{100 \times 2 \times 0.05 \times 0.04 \times \sin 90°} = \mathbf{0.5\ T}.$$

6. $\tau = NIB(\pi r^2) \sin\theta = 500 \times 1.0 \times 0.40 \times 3.14 \times (0.02)^2 \times \sin 30° = 0.1256 \simeq \mathbf{0.13\ Nm}.$

7. Proceed as in Exercise 4.25 on page 4.104.

4.20 MOVING COIL GALVANOMETER

23. *Describe the principle, construction and working of a pivoted-type moving coil galvanometer. Define its figure of merit.*

Moving coil galvanometer. *A galvanometer is a device to detect current in a circuit.* The commonly used moving coil galvanometer is named so because it uses a current-carrying coil that rotates (or moves) in a magnetic field due to the torque acting on it.

In a *D'Arsonval galvanometer*, the coil is suspended on a phosphor-bronze wire. It is highly sensitive and requires careful handling. In *Weston galvanometer*, the coil is pivoted between two jewellel bearings. It is rugged and portable though less sensitive, and is generally used in laboratories. The basic principle of both types of galvanometers is same.

Principle. *A current carrying coil placed in a magnetic field experiences a current dependent torque, which tends to rotate the coil and produces angular deflection.*

Construction. As shown in Fig. 4.94, a Weston (pivoted-type) galvanometer consists of a rectangular coil of fine insulated copper wire wound on a light non-magnetic metallic (aluminium) frame. The two ends of the axle of this frame are pivoted between two jewelled bearings. The motion of the coil is controlled by a pair of hair springs of phosphor-bronze. The inner ends of the springs are soldered to the two ends of the coil and the outer ends are connected to the binding screws. The springs provide the restoring torque and serve as current leads. A light aluminium pointer attached to the coil measures its deflection on a suitable scale.

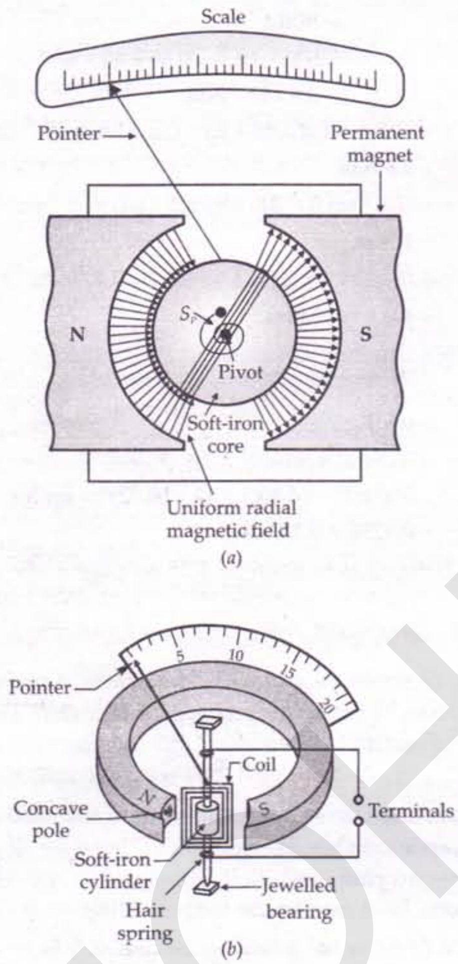


Fig. 4.94 (*a*) Top view (*b*) Front view of a pivoted-type galvanometer.

The coil is symmetrically placed between the cylindrical pole pieces of a strong permanent horse-shoe magnet.

A cylindrical soft iron core is mounted symmetrically between the concave poles of the horse-shoe magnet. This makes the lines of force pointing along the radii of a circle. Such a field is called a ***radial field***. The plane of a coil rotating in such a field remains parallel to the field in all positions, as shown in Fig. 4.94(*a*). Also, the soft iron cylinder, due to its high permeability, intensifies the magnetic field and hence increases the sensitivity of the galvanometer.

Theory and working. In Fig. 4.95(*a*), we have

I = current flowing through the coil $PQRS$

a, b = sides of the rectangular coil $PQRS$

$A = ab$ = area of the coil

N = number of turns in the coil.

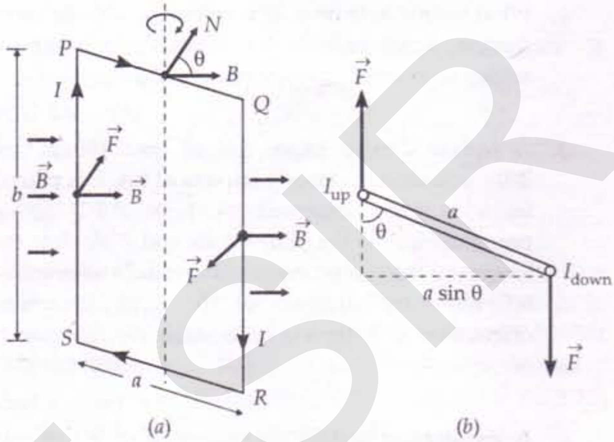


Fig. 4.95 (*a*) Rectangular loop *PQRS* in a uniform magnetic field. (*b*) Top view of the loop.

Since the field is radial, the plane of the coil always remains parallel to the field $\vec{B}$. The magnetic forces on sides PQ and SR are equal, opposite and collinear, so their resultant is zero. According to Fleming's left rule, the side PS experiences a normal inward force equal to $NIbB$ while the side QR experiences an equal normal outward force. The two forces on sides PS and QR are equal and opposite. They form a couple and exert a torque given by

$$\tau = \text{Force} \times \text{Perpendicular distance}$$

$$= NIbB \times a \sin 90° = NIB(ab) = NIBA$$

Here $\theta = 90°$, because the normal to the plane of coil remains perpendicular to the field $\vec{B}$ in all positions.

The torque τ deflects the coil through an angle α. A restoring torque is set up in the coil due to the elasticity of the springs such that

$$\tau_{restoring} \propto \alpha \quad \text{or} \quad \tau_{restoring} = k\alpha$$

where k is the ***torsion constant*** of the springs *i.e.*, torque required to produce unit angular twist. In equilibrium position,

Restoring torque = Deflecting torque

$$k\alpha = NIBA$$

or $$\alpha = \frac{NBA}{k} \cdot I$$

or $$\alpha \propto I$$

Thus the deflection produced in the galvanometer coil is proportional to the current flowing through it. Consequently, the instrument can be provided with a scale with equal divisions along a circular scale to indicate equal steps in current. Such a scale is called ***linear scale***.

Also, $I = \frac{k}{NBA} \cdot \alpha = G\alpha$

The factor $G = k / NBA$ is constant for a galvanometer and is called ***galvanometer constant*** or ***current reduction factor*** of the galvanometer.

Figure of merit of a galvanometer. *It is defined as the current which produces a deflection of one scale division in the galvanometer and is given by*

$$G = \frac{I}{\alpha} = \frac{k}{NBA}$$

4.21 SENSITIVITY OF A GALVANOMETER

24. *When is a galvanometer said to be sensitive ? Define current sensitivity and voltage sensitivity of a galvanometer. State the factors on which the sensitivity of a moving coil galvanometer depends. How can we increase the sensitivity of a galvanometer ?*

Sensitivity of a galvanometer. *A galvanometer is said to be sensitive if it shows large scale deflection even when a small current is passed through it or a small voltage is applied across it.*

Current sensitivity. *It is defined as the deflection produced in the galvanometer when a unit current flows through it.*

Current sensitivity, $I_S = \frac{\alpha}{I} = \frac{NBA}{k}$

Voltage sensitivity. *It is defined as the deflection produced in the galvanometer when a unit potential difference is applied across its ends.*

Voltage sensitivity, $V_S = \frac{\alpha}{V} = \frac{\alpha}{IR} = \frac{NBA}{kR}$

Clearly, voltage sensitivity $= \frac{\text{Current sensitivity}}{R}$

Factors on which the sensitivity of a moving coil galvanometer depends :

1. Number of turns N in its coil.
2. Magnetic field B.
3. Area A of the coil.
4. Torsion constant k of the spring and suspension wire.

Factors by which the sensitivity of a moving coil galvanometer can be increased :

1. *By increasing the number of turns N of the coil.* But the value of N cannot be increased beyond a certain limit because that will make the galvanometer bulky and increase its resistance R.
2. *By increasing the magnetic field B.* This can be done by using a strong horse-shoe magnet and placing a soft iron core within the coil.
3. *By increasing the area A of the coil.* However, increasing A beyond a certain limit will make the galvanometer bulky and unmanageable.
4. *By decreasing the value of torsion constant k.* The torsion constant k is made small by using suspension wire and springs of phosphor bronze.

25. *Give the advantages and disadvantages of using a moving coil galvanometer.*

Advantages of a moving coil galvanometer :

1. As the deflection of the coil is proportional to the current passed through it, so a linear scale can be used to measure the deflection.
2. A moving coil galvanometer can be made highly sensitive by increasing N, B, A and decreasing k.
3. As the coil is placed in a strong magnetic field of a powerful magnet, its deflection is not affected by external magnetic fields. This enables us to use the galvanometer in any position.
4. As the coil is wound over a metallic frame, the eddy currents produced in the frame bring the coil to rest quickly.

Disadvantages of a moving coil galvanometer :

1. The main disadvantage is that its sensitiveness cannot be changed at will.
2. All types of moving coil galvanometers are easily damaged by overloading. A current greater than that which the instrument is intended to measure will burn out its hair-springs or suspension.

For Your Knowledge

- If the radial field were not present in a moving coil galvanometer, for example, if the soft iron cylinder were removed, then the torque would be $NBAI \sin\theta$ and I would be proportional $\alpha / \sin\theta$. The scale would then be *non-linear* and difficult to calibrate or to read accurately.
- Phosphor-bronze is used for suspension or hair springs because of several reasons :
  1. It is a good conductor of electricity.
  2. It does not oxidise.
  3. It is perfectly elastic.
  4. It has very little elastic after effect.
  5. It is non-magnetic.
  6. Of all materials, it has the minimum value for restoring torque per unit twist *i.e.*, smallest torsion constant k.

Examples based on

Moving Coil Galvanometer and its Sensitivity

Formulae Used

1. In a moving coil galvanometer,

   Current, $I = \frac{k}{NBA} \cdot \alpha$

   Deflection produced, $\alpha = \frac{NBA}{k} \cdot I$

2. Figure of merit, $G = \frac{I}{\alpha} = \frac{k}{NBA}$

3. Current sensitivity, $I_S = \frac{\alpha}{I} = \frac{NBA}{k}$

4. Voltage sensitivity, $V_S = \frac{\alpha}{V} = \frac{\alpha}{IR} = \frac{NBA}{kR}$

Units Used

Current I is in ampere, area A in m^2, field B in tesla, angle α in degrees, torque τ in Nm, resistance R in ohm, potential difference V in volt, torsion constant k in Nm deg^{-1}.

Example 79. *A rectangular coil of area 5.0×10^{-4} m^2 and 60 turns is pivoted about one of its vertical sides. The coil is in a radial horizontal field of 90 G ('radial' here means the field lines are in the plane of the coil for any orientation). What is the torsional constant of the hair-springs connected to the coil if a current of 0.20 mA produces an angular deflection of 18° ?* [NCERT]

Solution. $B = 90 \text{ G} = 90 \times 10^{-4}$ T,

$A = 5.0 \times 10^{-4} \text{m}^2$, $I = 0.20 \text{ mA} = 0.20 \times 10^{-3}$ A,

$N = 60$, $\alpha = 18°$

Torsional constant of the hair spring is given by

$$k = \frac{NIBA}{\alpha}$$

$$= \frac{60 \times 0.2 \times 10^{-3} \times 90 \times 10^{-4} \times 5 \times 10^{-4}}{18} \text{ Nm deg}^{-1}$$

$$= \mathbf{3.0 \times 10^{-9} \text{ Nm deg}^{-1}}.$$

Example 80. *A rectangular coil having each turn of length 5 cm and breadth 2 cm is suspended freely in a radial magnetic field of induction 2.5×10^{-2} Wb m^{-2}, torsional constant of the suspension fibre is 1.5×10^{-8} Nm rad^{-1}. The coil deflects through an angle of 0.2 radian when a current of 2 μA is passed through it. Find the number of turns of the coil.*

Solution. $A = 5 \text{ cm} \times 2 \text{ cm} = 10 \times 10^{-4} \text{m}^2 = 10^{-3} \text{m}^2$

$B = 2.5 \times 10^{-2}$ Wb m^{-2}, $k = 1.5 \times 10^{-8}$ Nm rad^{-1}

$\theta = 0.2$ rad, $I = 2\ \mu\text{A} = 2 \times 10^{-6}$ A

As $\quad I = \frac{k}{NBA} \cdot \alpha$

$\therefore \quad N = \frac{k}{IBA} \cdot \alpha$

$$= \frac{1.5 \times 10^{-8} \times 0.2}{2 \times 10^{-6} \times 2.5 \times 10^{-2} \times 10^{-3}} = \mathbf{60.}$$

Example 81. *The coil of a moving coil galvanometer has an effective area of 5×10^{-2} m^2. It is suspended in a magnetic field of 2×10^{-2} Wb m^{-2}. If the torsional constant of the suspension fibre is 4×10^{-9} Nm deg^{-1}, find its current sensitivity in degree per-microampere.*

Solution. Here $N = 1$, $A = 5 \times 10^{-2}$ m^2,

$B = 2 \times 10^{-2}$ Wb m^{-2}, $k = 4 \times 10^{-9}$ Nm deg^{-1}

Current sensitivity

$$= \frac{NBA}{k} = \frac{1 \times 2 \times 10^{-2} \times 5 \times 10^{-2}}{4 \times 10^{-9}}$$

$$= 0.25 \times 10^6 \text{ deg A}^{-1}$$

$$= \mathbf{0.25 \text{ deg } \mu\text{A}^{-1}} \qquad [\because 1 \text{ A} = 10^6 \mu\text{A}]$$

Example 82. *A current of 200 μA deflects the coil of a moving coil galvanometer through 30°. What should be the current to cause the rotation through π/10 radian ? What is the sensitivity of the galvanometer ?*

Solution. Here $I_1 = 200\ \mu\text{A}$, $\theta_1 = 30°$,

$\theta_2 = \frac{\pi}{10}$ rad $= 18°$, $I_2 = ?$

$$I_1 = \frac{k}{NBA} \cdot \alpha_1 \quad \text{and} \quad I_2 = \frac{k}{NBA} \cdot \alpha_2$$

$\therefore \quad \frac{I_2}{I_1} = \frac{\alpha_2}{\alpha_1}$

or $\quad I_2 = \frac{\alpha_2}{\alpha_1} \cdot I_1 = \frac{18}{30} \times 200 = \mathbf{120\ \mu A}$

Current sensitivity

$$= \frac{\alpha_2}{I_2} = \frac{18 \text{ deg}}{120\ \mu\text{A}} = \mathbf{0.15 \text{ deg } \mu\text{A}^{-1}}.$$

Example 83. *A galvanometer needs 50 mV for a full scale deflection of 50 divisions. Find its voltage sensitivity. What must be its resistance if its current sensitivity is 1 division/ μA ?*

Solution. Voltage sensitivity,

$$V_S = \frac{\alpha}{V} = \frac{50 \text{ divisions}}{50 \text{ mV}}$$

$$= \frac{50 \text{ divisions}}{50 \times 10^{-3} \text{ V}} = \mathbf{10^3 \text{ div V}^{-1}}$$

Resistance of galvanometer,

$$R_g = \frac{I_S}{V_S} = \frac{1 \text{ div } \mu A^{-1}}{10^3 \text{ div } V^{-1}}$$

$$= \frac{10^6 \text{ div } A^{-1}}{10^3 \text{ div } V^{-1}} = \mathbf{1000\ \Omega}.$$

Example 84. *A moving coil meter has the following particulars : Number of turns, $N = 24$; Area of the coil, $A = 20 \times 10^{-3} m^2$; Magnetic field strength, $B = 0.20$ T; Resistance of the coil, $R = 14\ \Omega$.*

(*i*) *Indicate a simple way to increase the current sensitivity of the meter by 25%. (It is not easy to change A or B).*

(*ii*) *If in doing so, the resistance of the coil increases to $21\ \Omega$, is the voltage sensitivity of the modified meter greater or less than the original meter ?* **[NCERT]**

Solution. (*i*) Current sensitivity,

$$I_S = \frac{\alpha}{I} = \frac{NBA}{k}$$

Since it is easier to change N than k, A or B, so the current sensitivity can be increased by increasing N. To increase it by 25%, N should be increased from 24 to 30.

(*ii*) Voltage sensitivity,

$$V_S = \frac{\alpha}{V} = \frac{\alpha}{RI} = \frac{NBA}{kR}$$

As k, A, B are same in the two cases, we need to compare only N/R ratio.

For original meter, $\frac{N}{R} = \frac{24}{14} = 1.7$

For modified meter, $\frac{N}{R} = \frac{30}{21} = 1.4$

Thus the modified meter has less voltage sensitivity than the original meter. By increasing the number of turns, it has gained in current sensitivity but lost in voltage sensitivity.

Example 85. *To increase the current sensitivity of a moving coil galvanometer by 50%, its resistance is increased so that the new resistance becomes twice its initial resistance. By what factor does its voltage sensitivity change ?* **[CBSE OD 01]**

Solution. Current sensitivity,

$$I_S = \frac{\alpha}{I}$$

Voltage sensitivity,

$$V_S = \frac{\alpha}{V} = \frac{\alpha}{IR} = \frac{I_S}{R}$$

New current sensitivity,

$$I'_S = I_S + \frac{50}{100} I_S = \frac{3}{2} I_S$$

New voltage sensitivity,

$$V'_S = \frac{I'_S}{2R} = \frac{\frac{3}{2} I_S}{2R} = \frac{3}{4} V_S = 0.75\ V_S$$

Thus new voltage sensitivity becomes 75% of its initial value *i.e.*, it decreases by 25%.

Example 86. *The coil of a galvanometer is $0.02\ m \times 0.08\ m$. It consists of 200 turns of fine wire and is in a magnetic field of 0.2 tesla. The restoring torque constant of the suspension fibre is 10^{-6} Nm deg^{-1}. Assuming the magnetic field to be radial, (a) what is the maximum current that can be measured by this galvanometer, if the scale can accommodate 30° deflection ? (b) What is the smallest current that can be detected, if the minimum observable deflection is 0.1 degree ?* **[CBSE OD 13C]**

Solution. Here $A = 0.02 \times 0.08 \text{ m}^2 = 1.6 \times 10^{-3} \text{m}^2$

$N = 200$, $B = 0.2$ T, $k = 10^{-6}$ Nm deg^{-1}

(*a*) The maximum current (I_{max}) that can be measured is given by

$$NBAI_{max} = k\alpha_{max}$$

or

$$I_{max} = \frac{k\ \alpha_{max}}{NBA} = \frac{10^{-6} \times 30}{200 \times 0.2 \times 1.6 \times 10^{-3}} \text{ A}$$

$$= \mathbf{4.69 \times 10^{-4}\ A.}$$

(*b*) The smallest current (I_{min}) that can be detected is given by

$$I_{min} = \frac{k\ \alpha_{min}}{NBA}$$

$$= \frac{10^{-6} \times 0.1}{200 \times 0.2 \times 1.6 \times 10^{-3}} \text{ A} = \mathbf{1.56 \times 10^{-6}\ A.}$$

Problems For Practice

1. A rectangular coil of area 100 cm^2 and consisting of 100 turns is suspended in a magnetic field of 5×10^{-2} T. What current should be made to pass through it in order to keep equilibrium at an angle of 45° with the field ? Given that torsion constant of the suspension fibre is 10^{-8} Nm deg^{-1}. (**Ans.** 9×10^{-6} A)

2. The coil of a galvanometer consists of 250 turns of fine wire wound on a 2.0 cm × 1.0 cm rectangular frame. It is suspended in a uniform radial magnetic field of strength 2,000 G. A current of 10^{-4} A produces an angular deflection of 30° in the coil. Find the torsional constant of its suspension. (**Ans.** 1.9×10^{-6} Nm rad^{-1})

3. A moving coil galvanometer is placed in a radial magnetic field of 0.2 T. The galvanometer coil has 200 turns and area of $1.6 \times 10^{-4} \text{m}^2$. The torsion constant of the suspension fibre is 10^{-6} Nm deg^{-1}.

Determine the maximum current that can be measured by this galvanometer if its scale can accommodate a deflection of 45°. (Ans. 7×10^{-3} A)

4. The coil of moving coil galvanometer is 40 mm long and 25 mm wide. It has 100 turns and is suspended in a radial magnetic field of 10^{-2} T. If the suspension fibre has a torsional constant of 10^{-8} Nm deg^{-1}, find the current sensitivity of the moving coil galvanometer. (Ans. 0.1 deg μA^{-1})
5. A coil of a moving coil galvanometer twists through 45° when a current of 1 micro-ampere is passed through it. If the area of the coil is 10^{-5} m^2 and it has 1000 turns, find the magnetic field of the magnet of the galvanometer. The restoring torque per unit twist of the galvanometer coil is 10^{-4} Nm deg^{-1}. (Ans. 45 T)
6. The coil of a pivoted-type galvanometer has 50 turns and encloses an area of 6 m^2. The magnetic field in the region in which the coil swings is 0.01 T and is radial. The torsional constant of the hair spring is 1.0×10^{-8} Nm deg^{-1}. Find the angular deflection of the coil for a current of 1 mA. (Ans. 30°)
7. A rectangular coil of area 8×10^{-4} m^2 is suspended freely in a radial magnetic field of induction 2×10^{-2} Wb m^{-2}. When a current of 5 μA is passed through the coil, it deflects through 60°. The torsional constant of the suspension is 3.821×10^{-9} Nm rad^{-1}. Find the number of turns of the coil. (Ans. 50 turns)
8. A galvanometer needs 25 mV for a full scale deflection of 50 divisions. Find its voltage sensitivity. What must be its resistance if its current sensitivity is 1 div/20 μA ? (Ans. 2×10^3 div V^{-1}, 25 Ω)
9. If the current sensitivity of a moving coil galvanometer is increased by 20%, its resistance also increases by 1.5 times. How will the voltage sensitivity of the galvanometer be affected ? **[CBSE OD 99]** (Ans. Decreases by 20%)
10. Compare the current sensitivity and voltage sensitivity of the following moving coil galvanometers : M_1 and M_2 :

$N_1 = 30, A_1 = 1.5 \times 10^{-3}$ m^2,
$B_1 = 0.25$ T, $R = 20\,\Omega$.
$N_2 = 35, A_2 = 2.0 \times 10^{-3}$ m^2
$B_2 = 0.25$ T, $R = 30\,\Omega$.

You are given that the springs in the two meters have the same torsional constants. (Ans. 9 : 14, 27 : 28)

HINTS

1. $$I = \frac{k}{NBA} \cdot \alpha = \frac{10^{-8} \times 45}{100 \times 5 \times 10^{-2} \times 100 \times 10^{-4}}$$
$$= \mathbf{9 \times 10^{-6}\ A.}$$

2. $\alpha = 30° = \frac{\pi}{6}$ rad $= 0.523$ rad
$$k = \frac{NIBA}{\alpha} = \frac{250 \times 10^{-4} \times 0.2 \times 2 \times 10^{-4}}{0.523}$$
$$= \mathbf{1.9 \times 10^{-6}\ Nm\ rad^{-1}.}$$

3. $$I_{max} = \frac{k\,\alpha_{max}}{NBA} = \frac{10^{-6} \times 45}{200 \times 0.2 \times 1.6 \times 10^{-4}} = \mathbf{7 \times 10^{-3}\ A.}$$

4. Current sensitivity $= \frac{NBA}{k}$.

5. As $I = \frac{k}{NBA} \cdot \alpha$, so $B = \frac{k}{NIA} \cdot \alpha$

6. Use $\alpha = \frac{NBA}{k} \cdot I$.

7. Use $N = \frac{k}{IBA} \cdot \alpha$.

8. $$V_S = \frac{\alpha}{V} = \frac{50 \text{ divisions}}{25 \text{ mV}} = \frac{50 \text{ divisions}}{25 \times 10^{-3} \text{ V}}$$
$$= \mathbf{2 \times 10^3\ div\ V^{-1}}$$
$$R = \frac{I_S}{V_S} = \frac{1 \text{ div}/20\,\mu\text{A}}{2 \times 10^3 \text{ div V}^{-1}}$$
$$= \frac{\frac{1}{20} \times 10^6 \text{ div A}^{-1}}{2 \times 10^3 \text{ div V}^{-1}} = \mathbf{25\,\Omega.}$$

9. Current sensitivity, $I_S = \frac{\alpha}{I}$

Voltage sensitivity,
$$V_S = \frac{\alpha}{V} = \frac{\alpha}{IR} = \frac{I_S}{R}$$
New current sensitivity,
$$I'_S = I_S + \frac{20}{100} I_S = \frac{6}{5} I_S$$
New voltage sensitivity,
$$V'_S = \frac{I'_S}{1.5\,R} = \frac{\frac{6}{5} I_S}{1.5\,R} = \frac{4}{5} V_S = 0.8\,V_S$$
Thus the new voltage sensitivity becomes 80% of its initial value *i.e.*, it decreases by 20%.

10. Proceed as in Exercise 4.10 on page 4.99.

4.22 MEASUREMENT OF CURRENT AND VOLTAGE

Introduction. A galvanometer is a basic instrument for electrical measurements. It is a sensitive current detector. It produces a deflection proportional to the current flowing through it. It can be easily converted into an ammeter for measuring current and into voltmeter for measuring voltage.

Following essential requirements should be met while converting a galvanometer into ammeter or voltmeter :

1. Ammeter or voltmeter should be accurate, reliable and sensitive.
2. The use of these devices in a circuit must *not alter* the current in the circuit or the potential difference across any element in the circuit.

4.23 CONVERSION OF A GALVANOMETER INTO AN AMMETER

26. *Explain how can we convert a galvanometer into an ammeter of given range.*

Conversion of a galvanometer into an ammeter. *An ammeter is a device used to measure current through a circuit element.* To measure current through a circuit element, an ammeter is connected in *series* with that element so that the current which is to be measured actually passes through it. In order to ensure that its insertion in the circuit does not change the current, an ammeter should have zero resistance. So ammeter is designed to have very small effective resistance. In fact, *an ideal ammeter should have zero resistance.*

An ordinary galvanometer is a sensitive instrument. It gives full scale deflection with a small current of few microamperes. To measure large currents with it, a small resistance is connected in parallel with the galvanometer coil. The resistance connected in this way is called a ***shunt***. Only a small part of the total current passes through the galvanometer and remaining current passes through the shunt. The value of shunt resistance depends on the range of the current required to be measured.

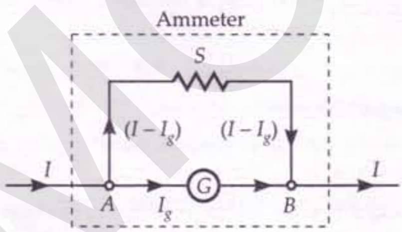


Fig. 4.96

Let G = resistance of the galvanometer

I_g = the current with which galvanometer gives full scale deflection

$0 - I$ = the required current range of the ammeter

S = shunt resistance

$I - I_g$ = current through the shunt.

As galvanometer and shunt are connected in parallel, so

P.D. across the galvanometer = P.D. across the shunt

$$I_g G = (I - I_g)\, S$$

or

$$S = \frac{I_g}{I - I_g} \times G$$

So by connecting a shunt of resistance S across the given galvanometer, we get an ammeter of desired range. Moreover,

$$I_g = \frac{S}{G+S} \times I$$

The deflection in the galvanometer is proportional to I_g and hence to I. So the scale can be graduated to read the value of current I directly.

Hence *an ammeter is a shunted or low resistance galvanometer.* Its effective resistance is

$$R_A = \frac{GS}{G+S} < S$$

27. *What is a shunt ? Mention its important uses.*

Shunt. *A shunt is a low resistance which is connected in parallel with a galvanometer (or ammeter) to protect it from strong currents.*

Uses of shunt :

1. To prevent a galvanometer from being damaged due to large current.
2. To convert a galvanometer into ammeter.
3. To increase the range of an ammeter.

For Your Knowledge

- Since an ammeter is a parallel combination of the galvanometer and the shunt resistance, so its resistance is even less than that of the shunt resistance. Moreover, $R_A << G$.
- Because of its very small resistance, an ammeter placed in a series circuit does not practically change the current in the circuit to be measured.
- The resistance of an ideal ammeter is zero.
- Higher the range of ammeter to be prepared from a given galvanometer, lower is the value of the shunt resistance required for the purpose.
- The ammeter of lower range has a higher resistance than the ammeter of higher range.
- The range of an ammeter can be increased but it cannot be decreased.

4.24 CONVERSION OF A GALVANOMETER INTO A VOLTMETER

28. *Explain how can we convert a galvanometer into a voltmeter of given range.*

Conversion of a galvanometer into a voltmeter. *A voltmeter is a device for measuring potential difference across any two points in a circuit.* It is connected in parallel with the circuit element across which the potential difference is intended to be measured. As a result, a small part of the total current passes through the voltmeter and so the current through the circuit element decreases. This decreases the potential difference required to be measured. To avoid this, the voltmeter should be designed to have very high resistance. In fact, *an ideal voltmeter should have infinite resistance.*

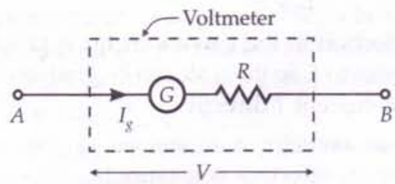


Fig. 4.97

A galvanometer can be converted into a voltmeter by connecting a high resistance in series with it. The value of this resistance is so adjusted that only current I_g which produces full scale deflection in the galvanometer, passes through the galvanometer.

Let

G = resistance of the galvanometer

I_g = the current with which galvanometer gives full scale deflection

$0 - V$ = required range of the voltmeter, and

R = the high series resistance which restricts the current to safe limit I_g.

$\therefore$ Total resistance in the circuit = $R + G$

By Ohm's law,

$$I_g = \frac{\text{Potential difference}}{\text{Total resistance}} = \frac{V}{R+G}$$

or $\quad R + G = \frac{V}{I_g} \quad$ or $\quad \boxed{R = \frac{V}{I_g} - G}$

So by connecting a high resistance R in series with the galvanometer, we get a voltmeter of desired range. Moreover, the deflection in the galvanometer is proportional to current I_g and hence to V. The scale can be graduated to read the value of potential difference directly.

Hence *a voltmeter is a high resistance galvanometer*. Its effective resistance is

$$\boxed{R_V = R + G >> G}$$

For Your Knowledge

- Since a voltmeter is a series combination of a galvanometer and a high resistance R, so its resistance is much higher than that of the galvanometer.
- An ideal voltmeter should have infinite resistance.
- A voltmeter is placed in parallel with the circuit element across which the voltage is to be measured. Because of its high resistance, it draws a very small current and hence the potential difference across the element remains practically unaffected.
- Higher the range of voltmeter to be prepared from a given galvanometer, higher is the value of series high resistance required for the purpose.
- The voltmeter of lower range has a lower resistance than the voltmeter of higher range.
- The range of voltmeter can be both increased or decreased.

Examples based on Conversion of Galvanometer into (*i*) Ammeter and (*ii*) Voltmeter, and Measurement of Current and Voltage

Formulae Used

1. For conversion of a galvanometer into ammeter, the shunt resistance,
$$R_s = \frac{I_g}{I - I_g} \times R_g \text{ ; Here } I_g = \frac{R_s}{R_g + R_s} \times I$$
2. Resistance of an ammeter, $R_A = \frac{R_g R_s}{R_g + R_s}$
3. For conversion of a galvanometer into a voltmeter, the value of high series resistance,
$$R = \frac{V}{I_g} - R_g \text{ ; Here } I_g = \frac{V}{R_g + R}$$
4. Resistance of a voltmeter, $R_V = R_g + R$
5. For a galvanometer, $I_g = nk$

   where n = no. of divisions on the galvanometer scale

   k = current required to produce deflection of one scale division or figure of merit of the galvanometer.

Units Used

All resistances are in ohm (Ω) and current in ampere (A).

Example 87. *A galvanometer with a coil of resistance 12.0 Ω shows full scale deflection for a current 2.5 mA. How will you convert the meter into :*

(*i*) *an ammeter of range 0 to 7.5 A,*

(*ii*) *a voltmeter of range 0 to 10.0 V ?*

Determine the net resistance of the meter in each case. When an ammeter is put in a circuit, does it read (slightly) less or more than the actual current in the original circuit ? When a voltmeter is put across a part of the circuit, does it read (slightly) less or more than the original voltage drop ? Explain. [NCERT ; CBSE D O5]

Solution. (*i*) *For conversion into ammeter* :

$$R_g = 12\ \Omega,\quad I_g = 2.5\text{ mA} = 0.0025\text{ A},\quad I = 7.5\text{ A}$$

$$R_s = \frac{I_g}{I - I_g} \times R_g = \frac{0.0025}{7.5 - 0.0025} \times 12$$

$$= \frac{2.5 \times 12 \times 10^{-3}}{7.4975} = \mathbf{4.0 \times 10^{-3}\ \Omega}$$

So by connecting a shunt resistance of $4.0 \times 10^{-3}\ \Omega$ in parallel with the galvanometer, we get an ammeter of range 0 to 7.5 A.

Net resistance R_A is given by

$$\frac{1}{R_A} = \frac{1}{12} + \frac{1}{4 \times 10^{-3}} = \frac{3001}{12}$$

or $$R_A = \frac{12}{3001}\ \Omega = \mathbf{4 \times 10^{-3}\ \Omega}$$

When an ammeter is put in a circuit, it reads slightly less than the actual current in the original circuit because a very small resistance is introduced in the circuit.

(*ii*) *For conversion into voltmeter* :

$$R_g = 12\ \Omega,\quad I_g = 2.5 \times 10^{-3}\text{ A},\quad V = 10\text{ V}$$

$$\therefore \quad R = \frac{V}{I_g} - R_g = \frac{10}{2.5 \times 10^{-3}} - 12$$

$$= 4000 - 12 = \mathbf{3988\ \Omega}$$

So by connecting a resistance of 3988 Ω in series with the galvanometer, we get a voltmeter of range 0 to 10 V.

Net resistance, $R_V = (3988 + 12)\ \Omega = \mathbf{4000\ \Omega}$

Because voltmeter draws small current for its deflection, so it reads slightly less than the original voltage drop.

Example 88. *A galvanometer with a scale divided into 100 equal divisions has a current sensitivity of 10 divisions per mA and a voltage sensitivity of 2 divisions per mV. What adoptions are required to read (i) 5 A for full scale and (ii) 1 division per volt ?* [IIT]

Solution. As the current sensitivity is 10 div per mA and there are 100 divisions on the scale, so current required for full scale deflection is

$$I_g = \frac{1}{10} \times 100\text{ mA} = 10\text{ mA} = 10 \times 10^{-3}\text{ A}$$

$$= 0.01\text{ A}$$

As voltage sensitivity is 2 div per mV, so voltage required for full scale deflection is

$$V_g = \frac{1}{2} \times 100\text{ mV} = 50\text{ mV} = 50 \times 10^{-3}\text{ V}$$

Galvanometer resistance,

$$R_g = \frac{V_g}{I_g} = \frac{50 \times 10^{-3}}{10 \times 10^{-3}} = 5\ \Omega$$

(*i*) *For conversion into an ammeter.* $I = 5$ A

$$R_s = \frac{I_g}{I - I_g} \times R_g = \frac{0.01}{5 - .01} \times 5 = \frac{5}{499}\ \Omega$$

So a shunt of $5/499\ \Omega$ should be connected across the galvanometer to read 5 A for full scale deflection.

(*ii*) *For conversion into voltmeter.* For reading 1 div per volt, the voltage range should be 100 V because there are 100 divisions.

$$\therefore \quad R = \frac{V}{I_g} - R_g = \frac{100}{0.01} - 5 = 10000 - 5 = 9995\ \Omega$$

So a resistance of 9995 Ω should be connected in series with the given galvanometer to read 1 div per volt.

Example 89. *An ammeter of resistance 0.80 Ω can measure currents upto 1.0 A. (i) What must be the shunt resistance to enable the ammeter to measure current upto 5.0 A ? (ii) What is the combined resistance of the ammeter and the shunt ?* [NCERT ; CBSE D 13]

Solution. The given ammeter can be regarded as the galvanometer.

$$\therefore \quad I_g = 1.0\text{ A},\quad R_g = 0.80\ \Omega$$

(*i*) Total current in the circuit, $I = 5.0$ A

The required shunt resistance,

$$R_s = \frac{I_g}{I - I_g} \times R_g = \frac{1.0}{5.0 - 1.0} \times 0.80 = \mathbf{0.20\ \Omega}.$$

(*ii*) The combined resistance R_A of the ammeter and the shunt is given by

$$\frac{1}{R_A} = \frac{1}{R_g} + \frac{1}{R_s} = \frac{1}{0.8} + \frac{1}{0.2} = \frac{1+4}{0.8} = \frac{25}{4}$$

or $$R_A = 4/25 = \mathbf{0.16\ \Omega}.$$

Example 90. *In the circuit (Fig. 4.98), the current is to be measured. What is the value of the current if the ammeter shown (a) is a galvanometer with a resistance $R_g = 60.00\ \Omega$;*

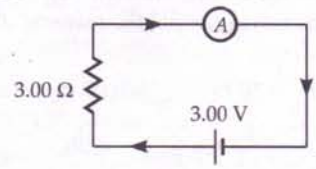


Fig. 4.98

(b) is a galvanometer described in (a) but converted to an ammeter by a shunt resistance $R_s = 0.02\,\Omega$; and (c) is an ideal ammeter with zero resistance ? [NCERT]

Solution. (*a*) Total resistance in the circuit

$$= R_g + 3\Omega = 60 + 3 = 63\,\Omega$$

Current, $I = \dfrac{3\text{ V}}{63\,\Omega} = \mathbf{0.048\ A.}$

(*b*) Resistance of the galvanometer converted to an ammeter is

$$R_A = \frac{R_g R_s}{R_g + R_s} = \frac{60 \times 0.02}{60 + 0.02} \simeq 0.02\,\Omega$$

Total resistance in the circuit

$$= R_A + 3\,\Omega = 0.02 + 3 = 3.02\,\Omega$$

Current, $I = \dfrac{3\text{ V}}{3.02\,\Omega} = \mathbf{0.99\ A.}$

(*c*) As the ideal ammeter has zero resistance, so

Current, $I = \dfrac{3.00\text{ V}}{3.00\,\Omega} = \mathbf{1.00\ A.}$

Example 91. *In a galvanometer there is a deflection of 10 divisions per mA. The internal resistance of the galvanometer is $60\,\Omega$. If a shunt of $2.5\,\Omega$ is connected to the galvanometer and there are 50 divisions in all, on the scale of galvanometer, what maximum current can this galvanometer read ?* [CBSE D 01C]

Solution. As the galvanometer has 50 divisions, current required to produce full scale deflection is

$$I_g = \frac{1}{10} \times 50\text{ mA} = 5\text{ mA} = 0.005\text{ A}$$

$$R_g = 60\,\Omega \quad \text{and} \quad R_s = 2.5\,\Omega$$

Let I be the maximum current that the galvanometer can read.

Then $I_g = \dfrac{R_s}{R_g + R_s} \times I$

or $I = \dfrac{(R_g + R_s)\, I_g}{R_s} = \dfrac{(60 + 2.5)\, 5}{2.5} = \mathbf{125\ mA.}$

Example 92. *A galvanometer having a resistance of $50\,\Omega$ gives a full deflection for a current of 0.05 A. Calculate the length of the shunt wire of 2 mm diameter required to convert the galvanometer to an ammeter reading current upto 5 A. Specific resistance for the material of the wire is $5 \times 10^{-7}\,\Omega$ m* [Punjab 96]

Solution. $R_g = 50\,\Omega,\quad I_g = 0.05\text{ A},\quad I = 5\text{ A}$

$$R_s = \frac{I_g}{I - I_g} \times R_g = \frac{0.05}{5 - 0.05} \times 50 = \frac{50}{99}$$

Now $R = \dfrac{50}{99}\,\Omega,\ \rho = 5 \times 10^{-7}\,\Omega\text{ m}, r = 1\text{ mm} = 10^{-3}\text{ m}$

As $R = \rho\dfrac{l}{A}$, so length of required shunt is

$$l = \frac{RA}{\rho} = \frac{50 \times \pi \times (10^{-3})^2}{99 \times 5 \times 10^{-7}}\text{ m}$$

$$= \frac{10 \times 3.142 \times 10}{99}\text{ m} = \mathbf{3.17\ m.}$$

Example 93. *A moving coil galvanometer when shunted with a resistance of $5\,\Omega$ gives a full scale deflection for 250 mA and when a resistance of $1050\,\Omega$ is connected in series, it gives a full scale deflection for 25 volt. Find the resistance of the galvanometer and the current required to produce a full scale deflection when it is used alone.*

Solution. With shunt, current required to produce full scale deflection is given by

$$I_g = \frac{R_s}{R_g + R_s} \cdot I$$

$$\therefore \quad I_g = \frac{5 \times 250 \times 10^{-3}}{R_g + 5} \quad \text{...(1)}$$

When a resistance R is connected in series,

$$I_g = \frac{V}{R_g + R} = \frac{25}{R_g + 1050} \quad \text{...(2)}$$

From equations (1) and (2),

$$\frac{1.25}{R_g + 5} = \frac{25}{R_g + 1050}$$

or $1.25\ R_g + 1312.5 = 25\ R_g + 125$

or $23.75\ R_g = 1187.5$

or $R_g = \dfrac{1187.5}{23.75} = \mathbf{50\ \Omega.}$

Example 94. *When a galvanometer having 30 divisions scale and $100\,\Omega$ resistance is connected in series to a battery of emf 3 V through a resistance of $200\,\Omega$, shows full scale deflection. Find the figure of merit of the galvanometer in μA.*

Solution. Here $n = 30,\ R_g = 100\,\Omega,\ \varepsilon = 3\text{ V}, R = 200\,\Omega, k = ?$

The current required to produce full scale deflection in the galvanometer is

$$I_g = \frac{\varepsilon}{R_g + R} = \frac{3}{100 + 200} = \frac{1}{100}\text{ A} = 10^4\ \mu\text{A}$$

As $I_g = nk$, therefore, the figure of merit is

$$k = \frac{I_g}{n} = \frac{10^4\ \mu\text{A}}{30\text{ divisions}} = \mathbf{333.3\ \mu A\ div^{-1}.}$$

Example 95. *The deflection produced in a galvanometer is reduced to 45 divisions from 55 when a shunt of 8 Ω is used. Calculate the resistance of the galvanometer.*

Solution. Without shunt, $I_g = I = 55k$

where k is the figure of merit of the galvanometer.

With shunt,

$$I'_g = (55 - 45)\,k = 10k$$

$$\therefore \quad \frac{I'_g}{I_g} = \frac{10k}{55k} = \frac{2}{11}$$

or $$I'_g = \frac{2}{11} I_g = \frac{2}{11} I$$

When shunt of 8 Ω is used,

$$I'_g = \frac{R_s}{R_g + R_s} \times I$$

or $$\frac{8}{R_g + 8} \times I = \frac{2}{11} I \quad \text{or} \quad 88 = 2R_g + 16$$

or $$R_g = \frac{72}{2} = \mathbf{36\ \Omega.}$$

Example 96. *A galvanometer of resistance 'G' can be converted into a voltmeter of range $(0 - V)$ volts by connecting a resistance 'R' in series with it. How much resistance will be required to change its range from 0 to $V/2$?* [CBSE OD 15C]

Solution. *In first case,* $R = \frac{V}{I_g} - G$

$$\therefore \quad I_g = \frac{V}{R + G}$$

Let R' be the required resistance to change the range from 0 to $V/2$. So *in second case,*

$$I_g = \frac{V/2}{R' + G}$$

$$\therefore \quad \frac{V}{R + G} = \frac{V/2}{R' + G}$$

or $$2R' + 2G = R + G$$

Hence, $$R' = \mathbf{\frac{R - G}{2}}.$$

Example 97. *A galvanometer can be converted into a voltmeter of certain range by connecting a resistance of 980 Ω in series with it. When the resistance is 470 Ω connected in series, the range is halved. Find the resistance of the galvanometer.*

Solution. The current for full scale deflection of a voltmeter is given by

$$I_g = \frac{V}{R_g + R}$$

In first case, $$I_g = \frac{V}{R_g + 980}$$

In second case, $$I_g = \frac{V/2}{R_g + 470}$$

$$\therefore \quad \frac{V}{R_g + 980} = \frac{V}{2\,(R_g + 470)}$$

or $$2R_g + 940 = R_g + 980$$

or $$R_g = \mathbf{40\ \Omega.}$$

Example 98. *A voltmeter reads 5.0 V at full scale deflection and is graded according to its resistance per volt at full scale deflection as 5000 Ω V^{-1}. How will you convert it into a voltmeter that reads 20 V at full scale deflection? Will it still be graded as 5000 ΩV^{-1}? Will you prefer this voltmeter to one that is graded as 2000 ΩV^{-1}?* [NCERT ; CBSE D 01C]

Solution. Resistance per volt is another way of specifying the current at full scale deflection. The grading of 5000 ΩV^{-1} at full scale deflection means that the current required for full scale deflection is

$$I_g = \frac{1}{5000}\ \text{A} = 0.2\ \text{mA}$$

In order to convert it into a voltmeter of range 0 to 20 V, a resistance R has to be connected in series with it. Then on applying an extra P.D. of 15 V (20 V − 5 V), the current through it is again 0.2 mA at full scale deflection.

$$\therefore \quad R \times 0.2 \times 10^{-3} = 15$$

or $$R = \frac{15}{0.2 \times 10^{-3}}\ \Omega = 75{,}000\ \Omega$$

Thus (*i*) to convert the given voltmeter (0 – 5 V range) into a voltmeter of range 0 to 20 V, a resistance of 75,000 Ω should be connected in series with the given meter.

(*ii*) Original resistance of voltmeter

$$= 5000\ \Omega V^{-1} \times 5\ V = 25{,}000\ \Omega$$

∴ Total resistance after conversion

$$= 25{,}000 + 75{,}000 = 100{,}000\ \Omega$$

Resistance per volt of new meter

$$= \frac{100{,}000}{20} = 5{,}000\ \Omega V^{-1}$$

i.e., it has the same grading as before.

(*iii*) The higher the 'resistance per volt' of the meter, the lesser is the current it draws from the circuit and the better it is. So this meter is more accurate than the one graded as 2000 Ω V^{-1}.

Example 99. *A galvanometer having 30 divisions has a current sensitivity of 20 μA/division. It has a resistance of*

$20\,\Omega$. *How will you convert it into an ammeter measuring upto 1 ampere ? How will you convert this ammeter into voltmeter reading upto 1 volt ?* [Roorkee 87]

Solution. Here $n = 30$, $R_g = 20\,\Omega$

Current sensitivity, $k = 20\,\mu A / \text{div}$

$\therefore$ Current required for full-scale deflection is

$$I_g = nk = 30 \times 20 = 600\,\mu A = 6 \times 10^{-4} A = 0.0006\ A$$

(*i*) *For conversion into ammeter* . $I = 1$ A

$$\therefore \quad R_s = \frac{I_g}{I - I_g} \times R_g = \frac{0.0006 \times 20}{1 - 0.0006}$$

$$= \frac{25 \times 6}{9994} = \mathbf{0.15\,\Omega}$$

i.e., a shunt of $0.15\,\Omega$ should be connected across the galvanometer.

(*ii*) *For conversion of resulting ammeter into voltmeter.* The resistance of the ammeter formed is

$$R'_g = \frac{R_g\, R_s}{R_g + R_s} = \frac{20 \times 0.15}{20 + 0.15} = 0.015\,\Omega$$

Current for full scale deflection, $I'_g = 1$ A

Voltage range, $V = 1$ V

$\therefore$ Required series resistance,

$$R = \frac{V}{I'_g} - R'_g = \frac{1}{1} - 0.015 = \mathbf{0.985\,\Omega}.$$

Example 100. *A voltmeter V of resistance* $400\,\Omega$ *is used to measure the potential difference across a* $100\,\Omega$ *resistor in the circuit shown in Fig. 4.99.*

(*i*) *What will be the reading on the voltmeter ?*

(*ii*) *Calculate the potential difference across* $100\,\Omega$ *resistor before the voltmeter is connected.* [CBSE D 98]

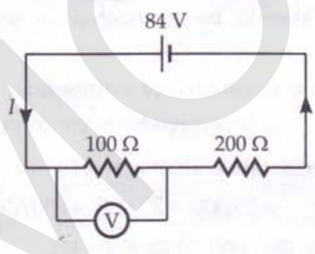


Fig. 4.99

Solution. (*i*) Resistance of the parallel combination of voltmeter V ($400\,\Omega$) and $100\,\Omega$ resistance

$$R' = \frac{400 \times 100}{400 + 100} = 80\,\Omega$$

Total resistance in the circuit,

$$R = R' + 200 = 80 + 200 = 280\,\Omega$$

Current in the circuit,

$$I = \frac{\mathcal{E}}{R} = \frac{84}{280} = \frac{3}{10}\ A$$

Reading on the voltmeter = P.D. across R'

$$= \frac{3}{10} \times 80 = \mathbf{24\ V}.$$

(*ii*) Total resistance before the voltmeter is connected $= 100 + 200 = 300\,\Omega$

$$\text{Current,}\quad I = \frac{84\ V}{300\,\Omega} = \frac{7}{25}\ A$$

$$\text{P.D. across } 100\,\Omega \text{ resistor} = \frac{7}{25} \times 100 = \mathbf{28\,\Omega}.$$

Example 101. *A d.c. supply of 120 V is connected to a large resistance X. A voltmeter of resistance* $10\ k\Omega$ *placed in series in the circuit reads 4 V. What is the value of X ? What do you think is the purpose in using a voltmeter, instead of an ammeter, to determine the large resistance X ?* [NCERT]

Solution. Current through voltmeter,

$$I = \frac{V}{R} = \frac{4\ V}{10 \times 10^3\,\Omega} = 4 \times 10^{-4} A$$

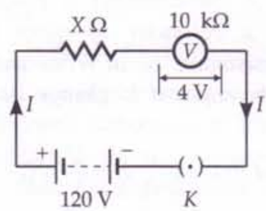


Fig. 4.100

$$\text{Also} \quad I = \frac{\text{total e.m.f.}}{\text{total resistance}}$$

$$\therefore \quad 4 \times 10^{-4} = \frac{120}{X + 10^4}$$

$$\text{or} \quad 4 \times 10^{-4} X + 4 = 120$$

$$\text{or} \quad X = \frac{116}{4 \times 10^{-4}}\,\Omega = 29 \times 10^4\,\Omega = \mathbf{290\ k\Omega}$$

As the current in the circuit is very small, the ammeter's reading will be too small to be measured accurately. This is an unusual use of voltmeter for measuring very high resistance.

Example 102. *(a) A battery of emf 9 V and negligible internal resistance is connected to a* $3\ k\Omega$ *resistor. The potential drop across a part of the resistor (between points A and B in Fig. 4.101) is measured by (i) a* $20\ k\Omega$ *voltmeter ; (ii) a* $1\ k\Omega$ *voltmeter. In (iii) both the voltmeters are connected across AB. In which case would you get the (1) highest, (2) lowest reading ?*

(b) Do your answers to this problem alter if the potential drop across the entire resistor is measured ? What if the battery has non-negligible resistance ? [NCERT]

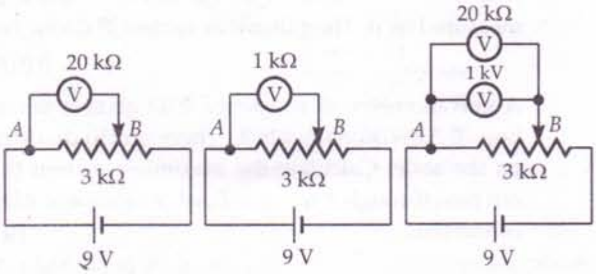


Fig. 4.101

Solution. (*a*) The voltmeter, which has maximum resistance, will draw minimum current and allow maximum current to flow through resistor *AB*. Consequently, there will be maximum potential difference across *AB*.

In case (*i*), resistance of voltmeter,

$$R_V = 20 \text{ k}\Omega$$

In case (*ii*), $R_V = 1 \text{ k}\Omega$

In case (*iii*), $\frac{1}{R_V} = \frac{1}{20} + \frac{1}{1} = \frac{1+20}{20}$

or $R_V = \frac{20}{21} \text{ k}\Omega$

(1) As the resistance of voltmeter is maximum in case (*i*), it will show maximum reading.

(2) As the resistance of voltmeter is minimum in case (*iii*), it will show lowest reading.

(*b*) In all cases, the voltmeter reading will be same if the battery has negligible internal resistance. But if the internal resistance is non-negligible, then the answers will be similar to those in (*a*).

Example 103. *You are given two resistors X and Y whose resistances are to be determined using an ammeter of resistance 0.5 Ω and a voltmeter of resistance 20 kΩ. It is known that X is in the range of a few ohms, while Y is the range of several thousand ohms. In each case, which of the two connections shown in Fig. 4.102 would you choose for resistance measurement ? Justify your answer quantitatively.* [NCERT]

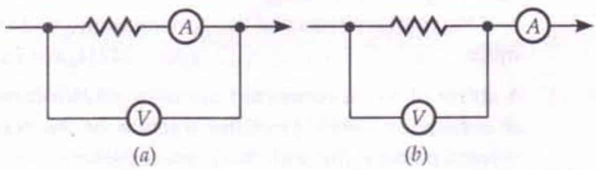


Fig. 4.102

Solution. In circuit (*a*), the voltmeter V will measure the sum of the potential drops across the resistance and the ammeter. The value of the resistance determined from these calculations will include the resistance of the ammeter. This will not be desirable if resistance is very small. So the circuit (*a*) is suitable only for measurement of large resistance *Y*.

In circuit (*b*), the ammeter will read the sum of currents flowing through the resistance and the voltmeter V. The value of the resistance obtained by these calculations will be less than the actual value. The difference will increase with the increase in the value of the resistance. So the circuit (*b*) is suitable only for the measurement of the small resistance *X*.

We can justify the above arguments quantitatively as follows :

(*i*) *Measurement of X. Let* $X = 5\,\Omega$. *In circuit (a)*, the ammeter reading is I_1 and the voltmeter reading is $I_1(X + 0.5)$.

$$\frac{\text{Voltmeter reading}}{\text{Ammeter reading}} = \frac{I_1(X+0.5)}{I_1} = \frac{I_1(5+0.5)}{I_1} = 5.5\,\Omega.$$

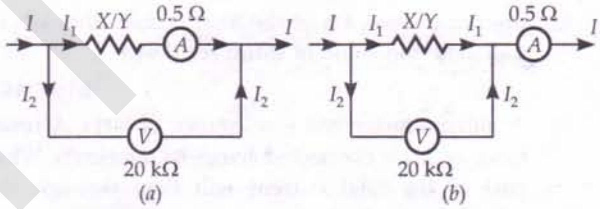


Fig. 4.103

With circuit (*a*), the error in the measurement of *X* is 0.5 Ω.

In circuit (*b*), the ammeter reading is *I* and the voltmeter reading is XI_1 ($= 20{,}000\, I_2$).

Clearly,

$$I_2 = \frac{XI_1}{20{,}000} = \frac{5}{20{,}000} I_1$$

$$\frac{\text{Voltmeter reading}}{\text{Ammeter reading}} = \frac{XI_1}{I} = \frac{5I_1}{I_1 + I_2}$$

$$= \frac{5I_1}{I_1 + \frac{5}{20{,}000} I_1}$$

$$= \frac{5 \times 20{,}000}{20{,}005} = 4.9987\,\Omega$$

With circuit (*b*), the error in the measurement of *X* is 0.0013 Ω. This error is much smaller than that obtained by using circuit (*a*). Hence for measuring a resistance of few ohms, the circuit (*b*) should be used.

(*ii*) *Measurement of Y*. Let $Y = 20{,}000\ \Omega$. In circuit (*a*), we get

$$\frac{\text{Voltmeter reading}}{\text{Ammeter reading}} = \frac{20{,}005\ I_1}{I_1} = 20{,}005\ \Omega$$

The error in the measurement of Y is $5\ \Omega$.

In circuit (*b*), we get

$$\frac{\text{Voltmeter reading}}{\text{Ammeter reading}} = \frac{20{,}000\ I_2}{I} = \frac{20{,}000\ I_2}{I_1 + I_2}$$

$$= \frac{20{,}000\ I_2}{I_2 + I_2} \quad [\because 20{,}000\ I_1 = 20{,}000\ I_2]$$

$$= 10{,}000\ \Omega$$

The error in the measurement of Y is $10{,}000\ \Omega$, which is much larger than error obtained by using circuit (*a*). Hence for measuring large resistance of several thousand ohms, the circuit (*a*) should be used.

Problems For Practice

1. A galvanometer has a resistance of $96\ \Omega$ and it is desired to pass 4% of the total current through it. Calculate the value of shunt resistance.
(Ans. $4\ \Omega$)

2. A galvanometer has a resistance of $50\ \Omega$. A resistance of $5\ \Omega$ is connected across its terminals. What part of the total current will flow through the galvanometer ? **[Haryana 01]**
(Ans. 1/11)

3. A galvanometer coil has a resistance of $30\ \Omega$ and the meter shows full scale deflection for the current of 2.0 mA. Calculate the value of resistance required to convert it into an ammeter of range 0 to 1 A. Also calculate the resistance of the ammeter. **[CBSE Sample Paper 98]**
(Ans. $0.06\ \Omega$ in parallel, $0.05988\ \Omega$)

4. How will you convert 1 mA full scale deflection meter of resistance 100 ohms into an ammeter to read 1 A (full scale deflection) and into a voltmeter to read 1 volt (full scale deflection). **[CBSE D 93C ; Punjab 97]**
(Ans. 0.1Ω in parallel, $900\ \Omega$ in series)

5. A moving coil galvanometer of resistance $10\ \Omega$ produces full scale deflection, when a current of 25 mA is passed through it. Describe showing full calculations, how will you convert the galvanometer into (*i*) a voltmeter reading upto 120 V and (*ii*) an ammeter reading upto 20 A. **[ISCE 94]**
(Ans. $4790\ \Omega$ in series, $0.0125\ \Omega$ in parallel)

6. A galvanometer of resistance $20\ \Omega$ gives a deflection of one division when a potential difference of 4 mV is applied across its terminals. Calculate the resistance of the shunt if the current of 10 A is to be measured by it. The galvanometer has 25 divisions.
(Ans. $0.01\ \Omega$)

7. A galvanometer of resistance $40\ \Omega$ gives a deflection of 5 divisions per mA. There are 50 divisions on the scale. Calculate the maximum current that can pass through it when a shunt resistance of $2\ \Omega$ is connected. **[IIT]**
(Ans. 210 mA)

8. It is intended to measure a maximum current of 25 A with an ammeter of range 2.5 A and resistance $0.9\ \Omega$. How will you do it ? What will be the combined resistance ? (Ans. 0.1Ω in parallel, $0.09\ \Omega$)

9. A galvanometer has a resistance of $30\ \Omega$ and a current of 2 mA is needed to give full scale deflection. What is the resistance needed and how is it to be connected to convert the galvanometer (*i*) into an ammeter of 0.3 A range and (*ii*) into a voltmeter of 0.2 V range ? **[Roorkee 92]**
$\left(\text{Ans. } (i)\ \frac{30}{149}\ \Omega \text{ in parallel } (ii)\ 70\ \Omega \text{ in series}\right)$

10. A galvanometer has a resistance of $100\ \Omega$. A difference of potential of 1.0 V between its terminals gives a full scale deflection. Calculate the shunt resistance which will enable the instrument to read upto 2 A. (Ans. $0.5\ \Omega$)

11. A resistance of $900\ \Omega$ is connected in series with a galvanometer of resistance $100\ \Omega$. A potential difference of 1 V produces a deflection of 100 divisions in the galvanometer. Find the figure of merit of the galvanometer. (Ans. 10^{-5} A div^{-1}.)

12. A galvanometer has a sensitivity of 60 divisions per ampere. When a shunt is used, its sensitivity becomes 10 divisions per ampere. What is the value of the shunt used if the resistance of the galvanometer is $20\ \Omega$? (Ans. $4\ \Omega$)

13. A galvanometer of resistance $3663\ \Omega$ gives full scale deflection for a certain current I_g. Calculate the resistance of the shunt which when joined to the galvanometer coil will result in 1/34 of the total current passing through the galvanometer. Also find the total resistance of the galvanometer and the shunt. (Ans. $111\ \Omega$, $107.7\ \Omega$)

14. A shunt of $6\ \Omega$ is connected across a galvanometer of resistance $294\ \Omega$. Find the fraction of the total current passing through the galvanometer.
(Ans. 1/50)

15. A galvanometer has current sensitivity of 5 divisions/mA and a voltage sensitivity of 2 divisions/mV. If the instrument has 30 divisions, how will you use it to measure (*i*) a current of 3 A and (*ii*) a voltage of 15 V ?

[Ans. (*i*) 0.005 Ω in parallel (*ii*) 2497.5 Ω in series]

16. It is required to pass only one-tenth of the main current through a galvanometer having a resistance of 27 Ω. Calculate the length of the wire of specific resistance $48 \times 10^{-6} \Omega$ cm and area of cross-section 0.2 mm^2 required to make a shunt for this purpose. (Ans. 1.25 m)

17. A galvanometer gives a full scale deflection with a current of 1 A. It is converted into ammeter of range 10 A. Find the ratio of the resistance of the ammeter to the resistance of the shunt used. (Ans. 9 : 10)

18. A galvanometer has a resistance of 8 Ω. It gives a full scale deflection for a current of 10 mA. It is to be converted into an ammeter of range 5 A. The only shunt resistance available is of 0.02 Ω, which is not suitable for this conversion. Find the value of resistance *R* that must be connected in series with the galvanometer (Fig. 4.104) to get ammeter of desired range. (Ans. 1.98 Ω)

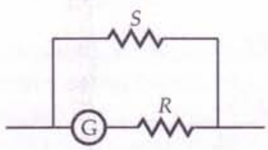


Fig. 4.104

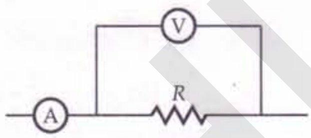


Fig. 4.105

19. The circuit shown in Fig. 4.105 is used to measure the resistance *R*. The ammeter reads 0.13 A and the voltmeter reads 117 V. The resistance of the ammeter is 0.015 Ω and that of the voltmeter is 9000 Ω. Find the value of *R*. (Ans. 1000 Ω)

20. The scale of a galvanometer is divided into 150 equal divisions. The galvanometer has the current sensitivity of 10 divisions per mA and the voltage sensitivity of 2 divisions per mV. How the galvanometer can be designed to read (*i*) 6 A $division^{-1}$ and (*ii*) 1 V per $division^{-1}$? **[Roorkee 99]**

[Ans. (*i*) 8.33×10^{-5} Ω in parallel (*ii*) 9995 Ω in series]

21. Two resistance coils of 100 Ω and 200 Ω respectively are connected in series across 100 V. A moving coil voltmeter of 200 Ω is connected in turn across each coil. What will it read in each case ?(Ans. 25 V, 50 V)

22. A battery of emf 12 V and internal resistance 1.2 Ω supplies a current through a coil of resistance 48 Ω. A voltmeter of resistance 72 Ω is used to measure the potential difference across the coil. What would be the reading on the voltmeter ? (Ans. 11.52 V)

HINTS

1. $R_s = \frac{I_g R_g}{I - R_g} = \frac{0.04\, I \times 96}{I - 0.04\, I} = 4 \Omega.$

2. As $R_s = \frac{I_g R_g}{I - I_g}$ or $\frac{I_g}{I} = \frac{R_s}{R_g + R_s} = \frac{5}{50 + 5} = \frac{1}{11}.$

3. Shunt resistance,

$$R_s = \frac{I_g}{I - I_g} \times R_g = \frac{0.002 \times 30}{1 - 0.002} = \frac{0.002 \times 30}{0.998} = \mathbf{0.06\, \Omega.}$$

Net resistance of the ammeter

$$= \frac{R_g R_s}{R_g + R_s} = \frac{30 \times 0.06}{30 + 0.06} = \frac{10}{167} = \mathbf{0.05988\, \Omega.}$$

4. (*i*) *For conversion into ammeter* :

$$R_g = 100\, \Omega, \quad I_g = 1\text{ mA} = 0.001\text{ A}, \quad I = 1\text{ A}$$

$$R_s = \frac{I_g}{I - I_g} \times R_g = \frac{0.001 \times 100}{1 - 0.001} = \frac{0.001 \times 100}{0.999}$$

$$\simeq \mathbf{0.1\, \Omega.}$$

(*ii*) *For conversion into voltmeter* :

$$R_g = 100\, \Omega, \quad I_g = 0.001\text{ A}, \quad V = 1\text{ V}$$

$$R = \frac{V}{I_g} - R_g = \frac{1}{0.001} - 100 = \mathbf{900\, \Omega.}$$

5. *For conversion into voltmeter* :

$$R = \frac{V}{I_g} - R_g = \frac{120}{25 \times 10^{-3}} - 10 = \mathbf{4790\, \Omega.}$$

For conversion into ammeter :

$$R_s = \frac{I_g\, R_g}{I - I_g} = \frac{25 \times 10^{-3} \times 10}{20 - 25 \times 10^{-3}} = \mathbf{0.0125\, \Omega.}$$

6. $I_g = \frac{V_g}{R_g} = \frac{4\text{ mV} \times 25}{20\, \Omega} = 5\text{ mA} = 5 \times 10^{-3}\text{ A}$

$$R_s = \frac{I_g R_g}{I - I_g} = \frac{5 \times 10^{-3} \times 20}{10 - 5 \times 10^{-3}} = \mathbf{0.01\, \Omega.}$$

7. $I_g = \frac{50}{5} = 10\text{ mA}, \; R_g = 40\, \Omega, \; R_s = 2\, \Omega$

∴ Maximum current,

$$I = \frac{R_g + R_s}{R_g} \times I_g = \frac{(40 + 2) \times 10}{2} = \mathbf{210\text{ mA}.}$$

8. To convert an ammeter of lower current range to higher current range, a shunt has to be connected across it. The shunt resistance is

$$R_s = \frac{I_g}{I - I_g} \times R_g = \frac{2.5 \times 0.9}{25 - 2.5} = 0.1\, \Omega$$

Combined resistance,

$$R = \frac{R_g\, R_s}{R_g + R_s} = \frac{0.9 \times 0.1}{0.9 + 0.1} = \mathbf{0.09\, \Omega.}$$

9. Proceed as in Example 88 on page 4.65.

10. $I_g = \frac{V_g}{R_g} = \frac{1}{100} = 0.01$ A

$$R_s = \frac{I_g}{I - I_g} \times R_g = \frac{0.01 \times 100}{2 - 0.01} = \mathbf{0.5\,\Omega}.$$

11. $R = \frac{V_g}{I_g} - R_g$ or $900 = \frac{1}{I_g} - 100$

or $I_g = \frac{1}{1000}$ A $= 10^{-3}$ A

Current sensitivity

$$= \frac{I_g}{n} = \frac{10^{-3}\text{ A}}{100\text{ div}} = \mathbf{10^{-5}\,A\,div^{-1}}.$$

12. Here $I \propto 60$ and $I_g \propto 10$

$\therefore \quad \frac{I_g}{I} = \frac{10}{60} = \frac{1}{6}$ and $R_g = 20\,\Omega$

But $\quad \frac{I_g}{I} = \frac{R_s}{R_g + R_s}$

$\therefore \quad \frac{1}{6} = \frac{R_s}{20 + R_s}$

or $6R_s = 20 + R_s$ or $R_s = \mathbf{4\,\Omega}.$

13. Here $I_g = \frac{I}{34}$ or $I = 34\,I_g$

$$\therefore \quad R_s = \frac{I_g}{I - I_g} \times R_g$$

$$= \frac{I_g \times 3663}{34\,I_g - I_g} = \mathbf{111\,\Omega}$$

Combined resistance,

$$R = \frac{R_g R_s}{R_g + R_s}$$

$$= \frac{3663 \times 111}{3663 + 111} = \mathbf{107.7\,\Omega}.$$

14. $\frac{I_g}{I} = \frac{R_s}{R_g + R_s} = \frac{6}{294 + 6} = \frac{1}{50}$ or 2% of the total current passes through the galvanometer.

15. Proceed as in Example 89 on page 4.65.

16. $R_s = \frac{I_g R_g}{I - I_g} = \frac{(I/10) \times 27}{I - I/10} = 3\,\Omega$

$$l = \frac{R_s A}{\rho} = \frac{3\,\Omega \times 0.2 \times 10^{-6}\text{ m}^2}{48 \times 10^{-8}\,\Omega\text{m}} = \mathbf{1.25\ m.}$$

17. $R_s = \frac{I_g R_g}{I - I_g} = \frac{1 \times R_g}{10 - 1} = \frac{R_g}{9}\,\Omega$

Resistance of the ammeter formed,

$$R_A = \frac{R_s R_g}{R_s + R_g} = \frac{(R_g/9)\,R_g}{R_g/9 + R_g} = \frac{R_g}{10}$$

$$\therefore \quad \frac{R_A}{R_s} = \frac{R_g/10}{R_g/9} = \mathbf{9:10.}$$

18. P.D. across the series combination of G and R

= P.D. across the shunt S

$I_g(R_g + R) = (I - I_g)\,R_s$

$0.01(8 + R) = (5 - 0.01) \times 0.02 = 4.99 \times 0.02$

or $\quad 8 + R = \frac{4.99 \times 0.02}{0.01} = 9.98$

or $\quad R = 9.98 - 8 = \mathbf{1.98\,\Omega.}$

19. Current through the voltmeter

$$= \frac{117\text{ V}}{9000\,\Omega} = 0.013\text{ A}$$

Current through the resistance

$$R = 0.13 - 0.013 = 0.117\text{ A}$$

Resistance,

$$R = \frac{117\text{ V}}{0.117\text{ A}} = \mathbf{1000\,\Omega.}$$

20. $I_g = \frac{1}{10} \times 150\text{ mA} = 15 \times 10^{-3}$ A

$V_g = \frac{1}{2} \times 150\text{ mV} = 75 \times 10^{-3}$ V

$R_g = \frac{V_g}{I_g} = \frac{75 \times 10^{-3}}{15 \times 10^{-3}} = 5\,\Omega.$

(*i*) Required current range, $I = 6 \times 150 = 900$ A

$$\therefore \quad R_s = \frac{I_g R_g}{I - I_g} = \frac{15 \times 10^{-3} \times 5}{900 - 15 \times 10^{-3}} = \mathbf{8.33 \times 10^{-5}\,\Omega.}$$

(*ii*) Required voltage range, $V = 1 \times 150 = 150$ V.

$$\therefore \quad R = \frac{V}{I_g} - R_g = \frac{150}{15 \times 10^{-3}} - 5 = \mathbf{9995\,\Omega.}$$

21. Proceed as in Example 100 on page 4.68.

22. Here $\varepsilon = 12$ V, $r = 1.2\,\Omega$, $R = 48\,\Omega$, $R_V = 72\,\Omega$

The combined resistance of the parallel combination of R and R_V will be

$$R_p = \frac{R \times R_V}{R + R_V} = \frac{48 \times 72}{48 + 72} = 28.8\,\Omega$$

Current in the circuit,

$$I = \frac{\varepsilon}{R_p + r} = \frac{12}{28.8 + 1.2} = 0.4\text{ A}$$

Reading of the voltmeter

$$= IR_p = 0.4 \times 28.8 = \mathbf{11.52\ V.}$$

GUIDELINES TO NCERT EXERCISES

4.1. *A circular coil of wire consisting of 100 turns each of radius 8.0 cm carries a current of 0.40 A. What is the magnitude of the magnetic field $\vec{B}$ at the centre of the coil ?*

Ans. Given $N = 100$, $r = 8\text{ cm} = 0.08\text{ m}$, $I = 0.40\text{ A}$

$$\therefore \quad B = \frac{\mu_0 NI}{2r} = \frac{4\pi \times 10^{-7} \times 100 \times 0.40}{2 \times 0.08}$$

$$= \pi \times 10^{-4} = \mathbf{3.1 \times 10^{-4}\ T.}$$

4.2. *A long straight wire carries a current of 35 A. What is the magnitude of the field $\vec{B}$ at a point 20 cm from the wire ?*

Ans. Here $I = 35\text{ A}$, $r = 20\text{ cm} = 0.20\text{ m}$,

$$\mu_0 = 4\pi \times 10^{-7}\text{ T mA}^{-1}$$

$$B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 35}{2\pi \times 0.20} = \mathbf{3.5 \times 10^{-5}\ T.}$$

4.3. *A long straight wire in the horizontal plane carries a current of 50 A in the north to south direction. Give the magnitude and direction of $\vec{B}$ at a point 2.5 m east of the wire.*

Ans. Here $I = 50\text{ A}$, $r = 2.5\text{ m}$

$$B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 50}{2\pi \times 2.5} = \mathbf{4 \times 10^{-6}\ T}$$

Applying right hand thumb rule, we find that the magnetic field will act in the vertically upward direction at the point 2.5 m east of the wire.

4.4. *A horizontal overhead power line carries a current of 90 A in an east to west direction. What is the magnitude and direction of magnetic field due to the current 1.5 m below the line ?*

Ans. Here $I = 90\text{ A}$, $r = 1.5\text{ m}$, $\mu_0 = 4\pi \times 10^{-7}\text{ Tm A}^{-1}$

$$B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 90}{2\pi \times 1.5}\text{ T} = \mathbf{1.2 \times 10^{-5}\ T}$$

Applying right hand thumb rule, we find that the direction of the field B will be towards south at a point below the power line.

4.5. *What is the magnitude of magnetic force per unit length on a wire carrying a current of 8 A and making an angle of 30° with the direction of a uniform magnetic field of 0.15 T ?*

Ans. Given $I = 8\text{ A}$, $\theta = 30°$, $B = 0.15\text{ T}$

As $\quad F = IlB\sin\theta$

Force per unit length,

$$f = \frac{F}{l} = IB\sin\theta$$

$$= 8 \times 0.15 \times \sin 30° = \mathbf{0.6\ Nm^{-1}}.$$

4.6. *A 3.0 cm wire carrying a current of 10 A is placed inside a solenoid perpendicular to its axis. The magnetic field inside the solenoid is given to be 0.27 T. What is the magnetic force on the wire ?*

Ans. Given $l = 3.0\,\text{cm} = 0.03\,\text{m}$, $I = 10\,\text{A}$,

$$\theta = 90°, \quad B = 0.27\,\text{T}$$

$$F = IlB\sin\theta = 10 \times 0.03 \times 0.27 \times \sin 90°$$
$$= \mathbf{8.1 \times 10^{-2}\ N}$$

The direction of the force is given by Fleming's left hand rule.

4.7. *Two long and parallel straight wires A and B carrying currents of 8.0 A and 5.0 A in the same direction are separated by a distance of 4.0 cm. Estimate the force on a 10 cm section of wire A.*

Ans. Force per unit length of each wire is

$$f = \frac{\mu_0 I_1 I_2}{2\pi r} = \frac{4\pi \times 10^{-7} \times 8 \times 5}{2\pi \times 4 \times 10^{-2}} = 2 \times 10^{-4}\,\text{Nm}^{-1}$$

Force on 10 cm section of wire A is

$$F = f\,l = 2 \times 10^{-4} \times 10 \times 10^{-2} = \mathbf{2 \times 10^{-5}\ N.}$$

4.8. *A closely wound solenoid 80 cm long has 5 layers of windings of 400 turns each. The diameter of the solenoid is 1.8 cm. If the current carried is 8.0 A, estimate the magnitude of $\vec{B}$ inside the solenoid near its centre.*

Ans. Number of turns per unit length of the solenoid is

$$n = \frac{\text{Number of turns per layer} \times \text{Number of layers}}{\text{Length of solenoid}}$$

$$= \frac{400 \times 5}{0.80} = 2500\,\text{m}^{-1}$$

Magnetic field inside the solenoid is

$$B = \mu_0 nI = 4\pi \times 10^{-7} \times 2500 \times 8 = 8\pi \times 10^{-3}\,\text{T}$$
$$= \mathbf{2.5 \times 10^{-2}\,T.}$$

4.9. *A square coil of the side 10 cm consists of 20 turns and carries a current of 12 A. The coil is suspended vertically and normal to the plane of the coil and makes an angle of 30° with the direction of a uniform horizontal magnetic field of magnitude 0.80 T. What is the magnitude of torque experienced by the coil ?*

Ans. Given $A = 0.10\,\text{m} \times 0.10\,\text{m} = 0.01\,\text{m}^2$, $N = 20$, $I = 12\,\text{A}$, $\theta = 30°$, $B = 0.80\,\text{T}$

Magnitude of torque is

$$\tau = NIBA\sin\theta$$
$$= 20 \times 12 \times 0.80 \times 0.01 \times \sin 30° = \mathbf{0.96\ Nm.}$$

4.10. *Two moving coil galvanometers M_1 and M_2 have the following particulars :*

$$R_1 = 10\,\Omega,\ N_1 = 30,\ A_1 = 3.6 \times 10^{-3}\,m^2,\ B_1 = 0.25\,T$$
$$R_2 = 14\,\Omega,\ N_2 = 42,\ A_2 = 1.8 \times 10^{-3}\,m^2,\ B_2 = 0.50\,T$$

The spring constants are identical for the two springs. Determine the ratio of (i) current sensitivity and (ii) voltage sensitivity of M_2 and M_1.

Ans. Let torsion constant for each meter = k

For a galvanometer, we have

$$NIBA = k\alpha$$

Its current sensitivity is defined as the deflection produced per unit current, *i.e.*,

$$\frac{\alpha}{I} = \frac{NBA}{k}$$

$$\therefore \quad \frac{\text{Current sensitivity of } M_2}{\text{Current sensitivity of } M_1} = \frac{N_2 B_2 A_2 / k}{N_1 B_1 A_1 / k} = \frac{N_2 B_2 A_2}{N_1 B_1 A_1}$$

$$= \frac{42 \times 0.50 \times 1.8 \times 10^{-3}}{30 \times 0.25 \times 3.6 \times 10^{-3}} = \frac{7}{5} = \mathbf{1.4}$$

Voltage sensitivity of a galvanometer is defined as the deflection produced per unit voltage, i.e.,

$$\frac{\alpha}{V} = \frac{\alpha}{IR} = \frac{NBA}{kR}$$

$$\frac{\text{Voltage sensitivity of } M_2}{\text{Voltage sensitivity of } M_1} = \frac{N_2 B_2 A_2 / kR_2}{N_1 B_1 A_1 / kR_1}$$

$$= \frac{N_2 B_2 A_2}{N_1 B_1 A_1} \times \frac{R_1}{R_2} = \frac{7}{5} \times \frac{10}{14} = \mathbf{1}$$

4.11. *In a chamber, a uniform magnetic field of 6.5 G (1 G = 10^{-4} T) is maintained. An electron is shot into the field with a speed of $4.8 \times 10^6\,ms^{-1}$ normal to the field. Explain why the path of the electron is a circle. Determine the radius of the circular orbit. Given that $e = 1.6 \times 10^{-19}\,C$, $m_e = 9.1 \times 10^{-31}\,kg$.*

Ans. The perpendicular magnetic field exerts a force on the electron perpendicular to its path. This force continuously deflects the electron from its path and makes it move along a circular path.

∴ Magnetic force on the electron = Centripetal force

$$e\,v\,B\sin 90° = \frac{m_e v^2}{r}$$

or

$$r = \frac{m_e v}{eB}$$

Now $B = 6.5\,\text{G} = 6.5 \times 10^{-4}\,\text{T}$, $v = 4.8 \times 10^6\,\text{ms}^{-1}$

$$\therefore \quad r = \frac{9.1 \times 10^{-31} \times 4.8 \times 10^6}{1.6 \times 10^{-19} \times 6.5 \times 10^{-4}} = 4.2 \times 10^{-2}\,\text{m} = \mathbf{4.2\ cm.}$$

4.12. *In Exercise 4.11, obtain the frequency of revolution of the electron in its circular orbit. Does the answer depend on the speed of the electron ? Explain.*

Ans. Frequency of revolution of the electron in its circular orbit,

$$f = \frac{eB}{2\pi m} = \frac{1.6 \times 10^{-19} \times 6.5 \times 10^{-4}}{2 \times 3.14 \times 9.1 \times 10^{-31}}$$

$$= 18.18 \times 10^6\,\text{Hz.} = \mathbf{18\ MHz.}$$

No, the frequency f does not depend on the speed v of the electron.

4.13. *(a) A circular coil of 30 turns and radius 8.0 cm carrying a current of 6.0 A is suspended vertically in a uniform* *horizontal magnetic field of magnitude 1.0 T. The field lines make an angle 60° with the normal to the coil. Calculate the magnitude of the counter torque that must be applied to prevent the coil from turning.*

(b) Would your answer change if the circular coil in (a) were replaced by a planar coil of some irregular shape that encloses the same area ? [CBSE OD 98C]

Ans. (*a*) $N = 30,\ r = 8.0\text{ cm} = 0.08\text{ m},\ I = 6.0\text{ A},$ $B = 1\text{ T},\ \theta = 60°$

Magnitude of counter torque

= Magnitude of deflecting torque

$= NIBA \sin\theta$

$= 30 \times 6 \times 1 \times (3.14 \times 0.08 \times 0.08) \sin 60°$

$= 30 \times 6 \times 3.14 \times 64 \times 10^{-4} \times 0.866 = \mathbf{3.1\ Nm}.$

(*b*) No, the answer would not change because the above formula for the torque is true for a planar loop of any shape.

4.14. *Two concentric circular coils X and Y of radii 16 cm and 10 cm respectively lie in the same vertical plane containing the north-south direction. Coil X has 20 turns and carries a current of 16 A ; coil Y has 25 turns and carries a current of 18 A. The sense of the current in X is anticlockwise, and in Y clockwise, for an observer looking at the coils facing west. Give the magnitude and direction of the net magnetic field due to the coils at their centre.*

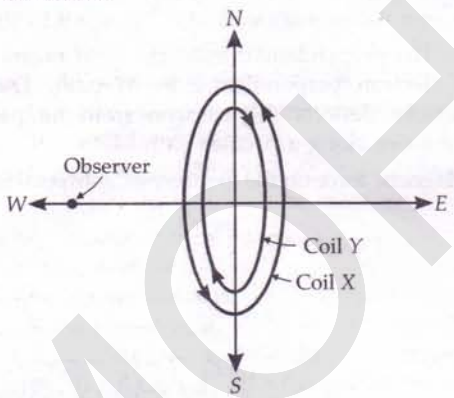


Fig. 4.160

Ans. **For coil X :** $r_x = 16\text{ cm} = 0.16\text{ m},\ N_x = 20,\ I_x = 16\text{ A}$

∴ Magnetic field at the centre of coil X is

$$B_x = \frac{\mu_0 I_x N_x}{2 r_x} = \frac{4\pi \times 10^{-7}}{2} \times \frac{16 \times 20}{0.16}\text{ T}$$

$$= 4\pi \times 10^{-4}\text{ T}$$

As the current in the coil X is anticlockwise, the field is directed towards east.

For coil Y : $r_y = 10\text{ cm} = 0.10\text{ m},\ N_x = 25,\ I = 18\text{ A}$

∴ Magnetic field at the centre of coil Y is

$$B_y = \frac{\mu_0 I_y N_y}{2 r_y} = \frac{4\pi \times 10^{-7}}{2} \times \frac{18 \times 25}{0.10}\text{ T} = 9\pi \times 10^{-4}\text{ T}$$

As the current in the coil Y is clockwise, the field B_y is directed towards west. Since $B_y > B_x$, therefore, the net field is directed ***towards west*** and its magnitude is

$$B = B_y - B_x = 5\pi \times 10^{-4} = \mathbf{1.6 \times 10^{-3}\ T.}$$

4.15. *A magnetic field of 100 G (1 G = 10^{-4} T) is required which is uniform in a region of linear dimension about 10 cm and area of cross-section about $10^{-3}\,m^2$. The maximum current-carrying capacity of a given coil of wire is 15 A and the number of turns per unit length that can be wound round a core is at most 1000 turns m^{-1}. Suggest some appropriate design particulars of a solenoid for the required purpose. Assume the core is not ferromagnetic.*

Ans. Here $B = 100\text{ G} = 10^{-2}\text{ T},\ I = 15\text{ A},$ $n = 1000\text{ turns m}^{-1}$

Magnetic field inside a solenoid,

$$B = \mu_0 n I$$

$$\therefore \quad nI = \frac{B}{\mu_0} = \frac{10^{-2}}{4\pi \times 10^{-7}} = 7955 \approx 8000$$

We may take $I = 10$ A, then $n = 800$

The solenoid may have length 50 cm and area of cross-section $5 \times 10^{-2}\,\text{m}^2$ (five times the given value) so as to avoid edge effects, etc.

4.16. *For a circular coil of radius R and N turns carrying current I, the magnitude of the magnetic field at a point on its axis at a distance x from its centre is given by,*

$$B = \frac{\mu_0 I R^2 N}{2(x^2 + R^2)^{3/2}}$$

(a) Show that this reduces to the familiar result for field at the centre of the coil.

(b) Consider two parallel co-axial circular coils of equal radius R, and number of turns N, carrying equal currents in the same direction, and separated by a distance R. Show that the field on the axis around the mid-point between the coils is uniform over a distance that is small as compared to R, and is given by

$$B = 0.72 \frac{\mu_0 NI}{R}, \text{ approximately}$$

Such an arrangement used to produce a nearly uniform magnetic field over a small region is known as Helmholtz coils.

Ans. (*a*) Given

$$B = \frac{\mu_0 I R^2 N}{2(x^2 + R^2)^{3/2}}$$

At the centre of the coil, $x = 0$, so

$$B = \frac{\mu_0 I R^2 N}{2R^3} = \frac{\mu_0 I N}{2R}$$

This is the standard result for the field at the centre of the coil.

(*b*) As shown in Fig. 4.161, consider a small region of length $2d$ about the midpoint O between the two coils.

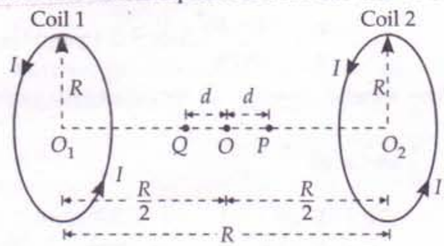


Fig. 4.161

Given $B = \dfrac{\mu_0 INR^2}{2(R^2 + x^2)^{3/2}}$

Therefore, the magnetic field at the point P due to coil 1,

$$B_1 = \frac{\mu_0 IR^2 N}{2\left[R^2 + \left(\frac{R}{2} + d\right)^2\right]^{3/2}}, \text{ acting along } PO_2$$

Magnetic field at the point P due to coil 2,

$$B_2 = \frac{\mu_0 IR^2 N}{2\left[R^2 + \left(\frac{R}{2} - d\right)^2\right]^{3/2}}, \text{ acting along } PO_2$$

Total magnetic field at the point P will be

$$B = B_1 + B_2$$

$$= \frac{\mu_0 IR^2 N}{2}\left[\frac{1}{\left(R^2 + \frac{R^2}{4} + d^2 + Rd\right)^{3/2}} + \frac{1}{\left(R^2 + \frac{R^2}{4} + d^2 - Rd\right)^{3/2}}\right]$$

$$= \frac{\mu_0 IR^2 N}{2}\left[\frac{1}{\left(\frac{5R^2}{4} + Rd\right)^{3/2}} + \frac{1}{\left(\frac{5R^2}{4} - Rd\right)^{3/2}}\right]$$

[Neglecting d^2, as $d << R$]

$$= \frac{\mu_0 IR^2 N}{2\left(\frac{5R^2}{4}\right)^{3/2}}\left[\frac{1}{\left(1 + \frac{4}{5}\frac{d}{R}\right)^{3/2}} + \frac{1}{\left(1 - \frac{4}{5}\frac{d}{R}\right)^{3/2}}\right]$$

$$= \frac{\mu_0 IN}{2R}\left(\frac{4}{5}\right)^{3/2}\left[\left(1 + \frac{4d}{5R}\right)^{-3/2} + \left(1 - \frac{4d}{5R}\right)^{-3/2}\right]$$

$$= \frac{\mu_0 IN}{2R}\left(\frac{4}{5}\right)^{3/2}\left[\left(1 - \frac{6d}{5R}\right) + \left(1 + \frac{6d}{5R}\right)\right]$$

[Expanding by binomial theorem and neglecting higher powers of d / R]

or $B \simeq 0.72 \dfrac{\mu_0 IN}{2R}$.

Magnetic field will also be same at the point Q. In fact, it will be uniform over the small region of length $2d$ around the midpoint O.

4.17. *A toroid has a core (non-ferromagnetic) of inner radius 25 cm and outer radius 26 cm around which 3500 turns of a wire are wound. If the current in the wire is 11 A, what is the magnetic field (a) outside the toroid (b) inside the core of the toroid (c) in the empty space surrounded by the toroid ?*

Ans. Here, $I = 11$ A, total number of turns = 3500

Mean radius of toroid,

$$r = \frac{25 + 26}{2} = 25.5 \text{ cm} = 25.5 \times 10^{-2} \text{ m}$$

Total length (circumference) of the toroid = $2\pi r$

$$= 2\pi \times 25.5 \times 10^{-2} = 51.0 \times 10^{-2}\ \pi \text{ m}$$

∴ Number of turns per unit length,

$$n = \frac{3500}{51.0 \times 10^{-2}\pi}$$

(*a*) The field outside the toroid is **zero.**

(*b*) The field inside the core of the toroid,

$$B = \mu_0 nI = 4\pi \times 10^{-7} \times \frac{3500}{51.0 \times 10^{-2}\pi} \times 11$$

$$= \mathbf{3.02 \times 10^{-2} T}.$$

(*c*) The field in the empty space surrounded by the toroid is also **zero.**

4.18. *Answer the following questions :*

(a) A magnetic field that varies in magnitude from point to point but has a constant direction (east to west) is set up in a chamber. A charged particle enters the chamber and travels undeflected along a straight path with constant speed. What can you say about the initial velocity of the particle ?

(b) A charged particle enters an environment of a strong and non-uniform magnetic field varying from point to point both in magnitude and direction and comes out of it following a complicated trajectory. Would its final speed equal the initial speed if it suffered no collisions with the environment ?

(c) An electron travelling west to east enters a chamber having a uniform electrostatic field in a north to south direction. Specify the direction in which a uniform magnetic field should be set up to prevent the electron from deflecting from its straight line path.

Ans. (*a*) The force on a charged particle moving in a magnetic field is given by

$$F = qvB \sin\theta$$

The force on a charged particle will be zero or the particle will remain undeflected if

$$\sin\theta = 0 \text{ or } \theta = 0°, 180°$$

i.e., initial velocity $\vec{v}$ is either *parallel or antiparallel* to $\vec{B}$.

(*b*) Yes, a magnetic field exerts force on a charged particle in a direction perpendicular to its direction of motion and hence does no work on it. So the charged particle will have its final speed equal to its initial speed.

(c) The electron travelling west to east experiences a force towards north due to the electrostatic field. It will remain undeflected if it experiences an equal force towards south due to the magnetic field. According to Fleming's left hand rule, the magnetic field must act in the *vertically downward direction.*

4.19. *An electron emitted by a heated cathode and accelerated through a potential difference of 2.0 kV, enters a region with uniform magnetic field of 0.15 T. Determine the trajectory of the electron if the field (i) is transverse to its initial velocity, (ii) makes an angle of 30° with the initial velocity.*

Ans. $V = 2.0\ \text{kV} = 2\times10^3\ \text{V}$, $B = 0.15\ \text{T}$,
$e = 1.6\times10^{-19}\ \text{C}$, $m = 9.1\times10^{-31}\ \text{kg}$

Potential difference V imparts kinetic energy to the electron given by

$$\frac{1}{2}mv^2 = eV$$

or, Velocity gained by electron,

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2\times1.6\times10^{-19}\times2\times10^3}{9.1\times10^{-31}}}\ \text{ms}^{-1}$$

$$= 2.65\times10^7\ \text{ms}^{-1}$$

(i) When field $\vec{B}$ is transverse to the initial velocity $\vec{v}$,

$$e\,v\,B\sin 90° = \frac{mv^2}{r}$$

$$\therefore \quad r = \frac{mv}{eB} = \frac{9.1\times10^{-31}\times2.65\times10^7}{1.6\times10^{-19}\times0.15}\ \text{m}$$

$$\simeq 10^{-3}\ \text{m} = \mathbf{1\ mm}.$$

Thus the electron follows a circular trajectory of radius 1 mm normal to the field B.

(ii) When field $\vec{B}$ makes an angle of 30° to the initial velocity $\vec{v}$,

$$v_\perp = v\sin 30° = 2.65\times10^7\times\frac{1}{2} = 1.33\times10^7\ \text{ms}^{-1}$$

$$v_\parallel = v\cos 30° = 2.65\times10^7\times0.866 = 2.3\times10^7\ \text{ms}^{-1}$$

The radius of the helical path is given by

$$r = \frac{mv_\perp}{eB} = \frac{mv\sin 30°}{eB} = \frac{9.1\times10^{-31}\times1.33\times10^7}{1.6\times10^{-19}\times0.15}$$

$$= 50.4\times10^{-5}\ \text{m} = \mathbf{0.50\ mm}.$$

4.20. *A magnetic field set up using Helmholtz coils is uniform in a small region and has a magnitude of 0.75 T. In the same region, a uniform electrostatic field is maintained in a direction normal to the common axis of the coils. A narrow beam of (single-species) charged particles all accelerated through 15 kV enters this region in a direction perpendicular to both the axis of the coils and the electrostatic field. If the beam remains undeflected when the electrostatic field is $9.0\times10^5\ Vm^{-1}$, make a simple guess as to what the beam contains. Why is the answer not unique ?*

Ans. $B = 0.75\ \text{T}$, $E = 9.0\times10^5\ \text{Vm}^{-1}$,
$V = 15\ \text{kV} = 15\times10^3\ \text{V}$

For undeflected beam, velocity of charged particles must be

$$v = \frac{E}{B} = \frac{9.0\times10^5}{0.75}\ \text{ms}^{-1} = 12\times10^5\ \text{ms}^{-1}$$

But the kinetic energy of the charged particles is given by

$$\frac{1}{2}mv^2 = qV$$

$$\therefore \quad \frac{q}{m} = \frac{1}{2}\cdot\frac{v^2}{V^2} = \frac{1}{2}\times\frac{(12\times10^5)^2}{15\times10^3}\ \text{C kg}^{-1}$$

$$= 4.8\times10^7\ \text{C kg}^{-1}$$

Now for deutrons,

$$\frac{q}{m} = \frac{1.6\times10^{-19}}{2\times1.67\times10^{-27}} = 4.8\times10^7\ \text{C kg}^{-1}$$

which means that the particles may be deutrons, each of which contains one proton and one neutron. The answer is not unique because we have determined only the ratio of charge to mass. Other possible answers are He^{2+} and Li^{3+}, etc.

4.21. *A straight horizontal conducting rod of length 0.45 m and mass 60 g is suspended by two vertical wires at its ends. A current of 5.0 A is set up in the rod through the wires.*

(a) What magnetic field should be set up normal to the conductor in order that the tension in the wires is zero ? [CBSE D 15C]

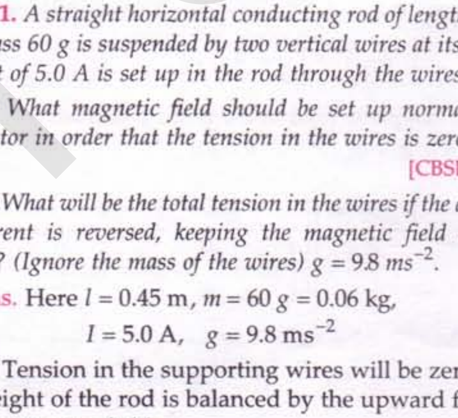
(b) What will be the total tension in the wires if the direction of current is reversed, keeping the magnetic field same as before ? (Ignore the mass of the wires) $g = 9.8\ ms^{-2}$.

Ans. Here $l = 0.45\ \text{m}$, $m = 60\ g = 0.06\ \text{kg}$,
$I = 5.0\ \text{A}$, $g = 9.8\ \text{ms}^{-2}$

(a) Tension in the supporting wires will be zero when the weight of the rod is balanced by the upward force IlB of the magnetic field.

i.e., $\quad IlB = mg$

$$\therefore \quad B = \frac{mg}{Il} = \frac{0.06\times9.8}{5\times0.45}\ \text{T} = \mathbf{0.26\ T}$$



According to Fleming's left hand rule, the magnetic field should be applied normally into the plane of paper so as to exert an upward magnetic force on the rod.

(b) If the direction of current is reversed, the magnetic force will act in the downward direction. Hence the total tension in the wires will be

$$T = 2\times\text{the weight of the rod}$$
$$= 2\times0.06\times9.8\ \text{N} = \mathbf{1.176\ N}.$$

4.22. *The wires which connect the battery of an automobile to its starting motor carry a current of 300 A (for a short time). What is the force per unit length between the wires if they are 70 cm long and 1.5 cm apart ? Is the force attractive or repulsive ?* [Haryana 01]

Ans. $I_1 = I_2 = 300$ A, $r = 1.5 \text{ cm} = 1.5 \times 10^{-2}$ m,
$l = 70 \text{ cm} = 0.70$ m

The force per unit length between the wires is

$$f = \frac{\mu_0 I_1 I_2}{2\pi r} = \frac{4\pi \times 10^{-7} \times 300 \times 300}{2\pi \times 1.5 \times 10^{-2}} \text{ Nm}^{-1} = \mathbf{1.2 \text{ Nm}^{-1}}$$

Total force between the wires,

$$F = f \times l = 1.2 \times 0.70 = \mathbf{0.84 \text{ N}}$$

As the currents in the two wires are in opposite directions, the force is repulsive.

4.23. *A uniform magnetic field of 1.5 T exists in a cylindrical region of radius 10.0 cm, its direction being parallel to the axis along east to west. A wire carrying current of 7.0 A in the north to south direction passes through this region. What is the magnitude and direction of the force on the wire if*

(i) the wire intersects the axis,

(ii) the wire is turned from N-S to north east or north west direction,

(iii) the wire in the N-S direction is lowered from the axis by a distance of 6.0 cm ?

Ans. Here $B = 1.5$ T, $I = 7.0$ A

(*i*) As shown in Fig. 4.163,

length of wire in cylindrical region

= diameter AB of cylindrical region

= 20 cm = 0.20 m

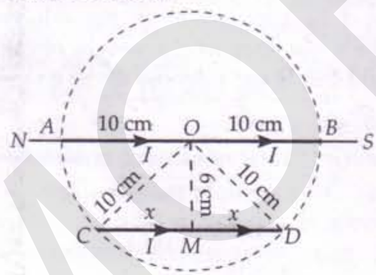


Fig. 4.163

As the wire lies in *N-S* direction and field acts along *E-W* direction, so $\theta = 90°$

∴ Force on wire,

$$F = IBl \sin\theta = 7 \times 1.5 \times 0.20 \times 1 = \mathbf{2.1 \text{ N}}$$

By Fleming's left hand rule, this force acts in the vertically downward direction.

(*ii*) When the wire turns from *N-S* to *N-E* or *N-W* direction, suppose it makes angle θ with field B, as shown in Fig. 4.164. Then length of wire in magnetic field, $A'B' = l'$ say.

Clearly $\frac{l}{l'} = \sin\theta$ or $l' = \frac{l}{\sin\theta}$

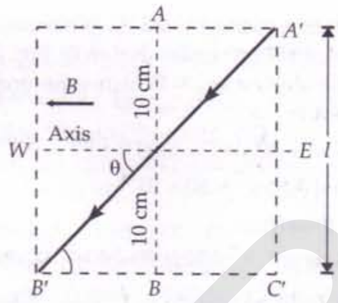


Fig. 4.164

Force on wire,

$$F = I l' \sin\theta = I \cdot \frac{l}{\sin\theta} \cdot B \sin\theta = IlB = 2.1 \text{ N}$$

This force acts in the vertically downward direction.

(*iii*) As shown in Fig. 4.159, when the wire is lowered by 6.0 cm, length of the wire in the magnetic field = $2x$

But $x = \sqrt{10^2 - 6^2} = 8 \text{ cm} = 0.08$ m

∴ $2x = 0.16, \quad \theta = 90°$

Force on wire, $F = IlB = 7 \times 0.16 \times 1.5 = \mathbf{1.68 \text{ N}}$

This force also acts in the vertically downward direction.

4.24. *A uniform magnetic field of 3000 G is established along the positive Z direction. A rectangular loop of sides 10 cm*

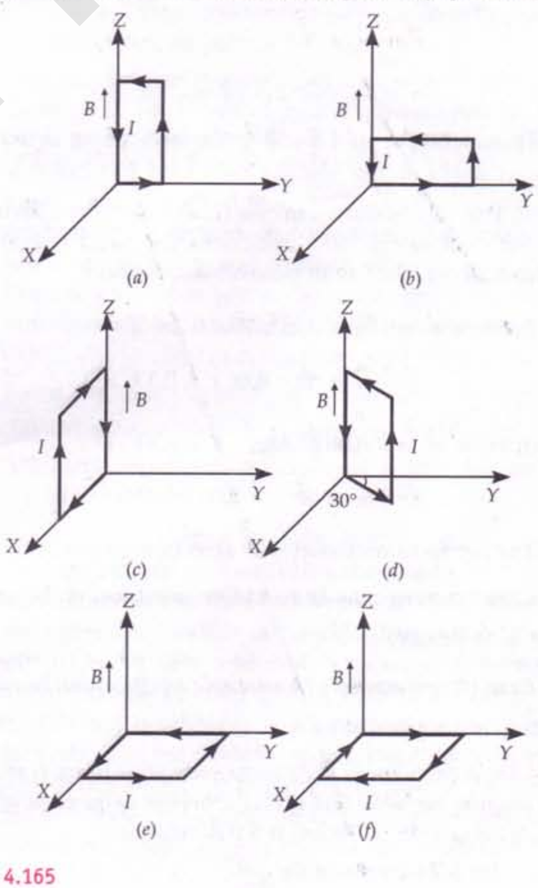


Fig. 4.165

and 5 cm carries a current of 12 A. What is the torque on the loop in the different cases shown in Fig. 4.165 ? What is the force on each case ? Which case corresponds to stable equilibrium ?

Ans. Here $B = 3000\,G = 3000 \times 10^{-4} = 0.3$ T,

$A = 10 \times 5 = 50\,cm^2 = 50 \times 10^{-4} m^2$, $I = 12$ A

Magnetic moment,

$$m = I\,A = 12 \times 50 \times 10^{-4} = 0.06\ Am^2$$

We apply right hand rule to various current loops to decide the direction of $\vec{m}$.

(*a*) Here $\vec{m} = 0.06\,\hat{i}$ Am^2, $B = 0.3\,\hat{k}$ T

$$\therefore \quad \vec{\tau} = \vec{m} \times \vec{B}$$

$$= 0.06\,\hat{i} \times 0.3\,\hat{k} = -1.8 \times 10^{-2}\,\hat{j}\ Nm$$

Thus a ***torque of 1.8 × 10^{-2} Nm acts along negative Y-axis.***

(*b*) Here $\vec{m} = 0.06\,\hat{i}$ Am^2, $\vec{B} = 0.3\,\hat{k}$ T

Clearly, $\vec{m}$ and $\vec{B}$ are same as in case (*a*). In this case also, a ***torque of 1.8 × 10^{-2} Nm acts along negative Y-axis.***

(*c*) Here $\vec{m} = -0.06\,\hat{j}$ Am^2, $\vec{B} = 0.3\,\hat{k}$ T

$$\vec{\tau} = \vec{m} \times \vec{B}$$

$$= -0.06\,\hat{i} \times 0.3\,\hat{k} = -1.8 \times 10^{-2}\,\hat{i}\ Nm$$

Thus a ***torque of 1.8 × 10^{-2} Nm acts along negative X-axis.***

(*d*) This case is similar to case (*c*). But here the direction of the torque is 60° anticlockwise with negative X-direction *i.e.*, 240° with positive X-direction.

(*e*) Here $\vec{m} = 0.06\,\hat{k}$ Am^2, $\vec{B} = 0.3\,\hat{k}$ T

$$\therefore \quad \vec{\tau} = \vec{m} \times \vec{B} = 0.06\,\hat{k} \times 0.3\,\hat{k} = \mathbf{0}.$$

(*f*) Here $\vec{m} = -0.06\,\hat{k}$ Am^2, $\vec{B} = 0.3\,\hat{k}$ T

$$\therefore \quad \vec{\tau} = \vec{m} \times \vec{B} = -0.06\,\hat{k} \times 0.3\,\hat{k} = \mathbf{0}.$$

The net force on the loop is zero in each case.

Case (e) corresponds to stable equilibrium, because here $\vec{m}$ is parallel to $\vec{B}$.

Case (f) corresponds to unstable equilibrium, because here $\vec{m}$ is antiparallel to $\vec{B}$.

4.25. *A circular coil of 20 turns and radius 10 cm is placed in a uniform magnetic field of 0.10 T normal to the plane of the coil. If the current in the coil is 5.0 A, what is the*

(*a*) *total torque on the coil,*

(*b*) *total force on the coil,*

(*c*) *average force on each electron in the coil due to the magnetic field ?*

(The coil is made of copper wire of cross-sectional area $10^{-5} m^2$, and the free electron density in copper is given to be about ($10^{29} m^{-3}$).

Ans. $N = 20$, $r = 10\,cm = 0.10$ m, $B = 0.10$ T,

$I = 5.0$ A, $\theta = 0°$

(*a*) Torque on the coil,

$$\tau = NIBA \sin\theta = \mathbf{0} \qquad [\because \theta = 0°]$$

(*b*) Magnetic forces on the opposite arms of coil are equal and opposite, and act in the same plane ; hence the total force on the coil is zero.

(*c*) Force on each electron is

$$F = evB = \frac{BI}{nA} \qquad [\because I = enAv]$$

For given wire,

$$n = 10^{29}\,m^{-3},\ A = 10^{-5}\,m^2$$

$$\therefore \quad F = \frac{0.1 \times 5}{10^{29} \times 10^{-5}}\ N = \mathbf{5 \times 10^{-25}\ N.}$$

4.26. *A solenoid 60 cm long and of radius 4.0 cm has 3 layers of windings of 300 turns each. A 2.0 cm long wire of mass 2.5 g lies inside the solenoid near its centre normal to its axis ; both the wire and the axis of the solenoid are in the horizontal plane. The wire is connected through two leads parallel to the axis of the solenoid to an external battery which supplies a current of 6.0 A in the wire. What value of current (with appropriate sense of circulation) in the windings of the solenoid can support the weight of the wire ? $g = 9.8 ms^{-2}$.*

Ans. Let I be the current in the windings of the solenoid which can support the weight of the wire. The magnetic field inside the solenoid along its axis will be

$$B = \mu_0 nI$$

Here,

$$n = \frac{\text{Total number of turns}}{\text{Length of the solenoid}}$$

$$= \frac{300 \times 3}{60 \times 10^{-2}} = 1500\ \text{turns}\ m^{-1}$$

$$\therefore \quad B = 4\pi \times 10^{-7} \times 1500 \times I$$

$$= 6\pi \times 10^{-4} I\ \text{tesla}$$

This field acts perpendicular to the current carrying wire, therefore, the magnetic force on the wire will be

$$F = I'lB$$

$$= 6 \times (2 \times 10^{-2}) \times 6\pi \times 10^{-4}\ I\ \text{newton}$$

The current I would support the wire if the above force equals the weight of the wire,

i.e., $6 \times 2 \times 10^{-2} \times 6\pi \times 10^{-4} I = 2.5 \times 10^{-3} \times 9.8$

or
$$I = \frac{2.5 \times 10^{-3} \times 9.8}{72 \times 3.14 \times 10^{-6}}\ A = \mathbf{108.3\ A.}$$

4.27. *A galvanometer coil has a resistance of 12 Ω and meter shows full scale deflection for a current of 3 mA. How will you convert the meter into a voltmeter of range 0 to 18 V ?*

Ans. Here $R_g = 12\,\Omega$, $I_g = 3\text{ mA} = 3 \times 10^{-3}\text{ A}$, $V = 18\text{ V}$

$$R = \frac{V}{I_g} - R_g = \frac{18}{3 \times 10^{-3}} - 12$$

$$= 6000 - 12 = \mathbf{5988\,\Omega}$$

By connecting a resistance of 5988 in series with the given galvanometer, we get a voltmeter of range 0 to 18 V.

4.28. *A galvanometer has a resistance of 15 Ω and the meter shows full scale deflection for a current of 4 mA. How will you convert the meter into an ammeter of range 0 to 6 A ?*

Ans. Here $R_g = 15\,\Omega$, $I_g = 4\text{ mA} = 0.004\text{ A}$, $I = 6\text{ A}$

$$R_s = \frac{I_g}{I - I_g} \times R_g = \frac{0.004}{6 - 0.004} \times 15$$

$$= 0.010\,\Omega = \mathbf{10\ m\,\Omega}$$

By connecting a shunt of resistance 10 mΩ across the given galvanometer, we get an ammeter of range 0 to 6 A.

TYPE A : VERY SHORT ANSWER QUESTIONS (1 mark each)

1. State Oersted's observation.
2. State Biot-Savart's law. [ISCE 94]
3. Mathematically, Biot-Savart law may be expressed as

$$dB = K . \frac{Idl \sin \theta}{r^2}$$

Write the value of K in SI units.
4. What is the SI unit of μ_0 ?
5. What is the value of $4\pi / \mu_0$?
6. State the rule that is used to find the direction of magnetic field acting at a point near a current carrying straight conductor. [CBSE 98 ; Pb. 97, 98]
7. Write an expression for the magnetic field produced by an infinitely long straight wire carrying a current I, at a short distance a from itself. [ISCE 98]
8. Show the magnetic lines of force around a straight current carrying conductor. [Punjab 97C]
9. What is the nature of the magnetic field associated with the current in a straight conductor ?
10. Where is the magnetic field due to current through circular loop uniform ?
11. Where is the magnetic field of a current element (*i*) minimum and (*ii*) maximum ?
12. Figure 4.166 shows a circular loop carrying a current I. Show the direction of the magnetic field with the help of lines of force. [CBSE D 04]

Fig. 4.166

13. Which physical quantity has the unit Wb m^{-2} ? Is it a scalar or a vector quantity ? [CBSE D 04]
14. An electric current is flowing due south along a power line. What is the direction of the magnetic field at a point (*a*) above it and (*b*) below it ?
15. How does a current carrying coil behave like a bar magnet ? [CBSE D 11]
16. Draw the magnetic field lines due to a current carrying loop. [CBSE D 13C]
17. How much is the flux density B at the centre of a long solenoid ? [ISCE 95, 97]
18. What is a toroid ?
19. What is magnetic Lorentz force ? [Punjab 01]
20. Write the expression, in a vector form, for the Lorentz magnetic force $\vec{F}$ due to a charge moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$. What is the direction of the magnetic force ? [CBSE D 14]
21. A particle of charge q moves with a velocity v at an angle θ to a magnetic field B. What is the force experienced by the particle ? [ISCE 93 ; CBSE F 91]
22. What is the force experienced by a stationary charge in a magnetic field ? [Himachal 02]
23. What is the work done by magnetic field on a moving charge ? [Haryana 94]
24. Write down the expression for the Lorentz force on a charged particle. [Himachal 99 ; Punjab 99, 99C]
25. What is the force on a charge moving along the direction of the magnetic field ? [CBSE D 94C]
26. State Fleming's left hand rule. [Pb 01 ; CBSE D 94C]
27. An electron beam is moving vertically downwards. If it passes through a magnetic field which is

directed from south to north in a horizontal plane, then in which direction the beam would be deflected ? [CBSE D 96C]

28. What will be the path of a charged particle moving perpendicular to a uniform magnetic field ? [CBSE D 93]
29. What will be the path of a charged particle moving in a uniform magnetic field at any arbitrary angle ? [CBSE F 93]
30. What will be the path of a charged particle moving along the direction of a uniform magnetic field ? [CBSE OD 95]
31. When a charged particle moving with a velocity $\vec{v}$ is subjected to a magnetic field $\vec{B}$ the force acting on it is non-zero. Would the particle gain any energy ? [CBSE F 13]
32. An electron with speed v enters at right angle in a region of uniform magnetic field B. Write the expression for the radius of the path it follows. [CBSE F 1995]
33. An electron beam projected along $+X$-axis, experiences a force due to a magnetic field along the $+Y$-axis. What is the direction of the magnetic field ? [CBSE D 05]
34. An electron and a proton moving with the same speed enter the same magnetic field region at right angles to the direction of the field. For which of the two particles will the radius of circular path be smaller ? [CBSE OD 98]
35. A charged particle moving in a uniform magnetic field penetrates a layer of lead and thereby loses one-half of its kinetic energy. How does the radius of curvature of its path change ? [CBSE F 10C]
36. Write the condition under which an electron will move undeflected in the presence of crossed electric and magnetic fields. [CBSE F 13 ; OD 14C]
37. A straight conductor AB of a circuit lies along the X-axis from $x = -a/2$ to $x = +a/2$ and carries a current I. What is the magnetic field due to this conductor AB at a point $x = +a$?
38. State the principle of a cyclotron. [Punjab 01]
39. Does the time spent by a proton inside the dees of a cyclotron depend on (*i*) the speed of the proton and (*ii*) the radius of its circular path ?
40. In a field, the force experienced by charge depends upon its velocity and becomes zero, when it is at rest. Is the field electric or magnetic in nature ?
41. In a field, the force experienced by a charge depends only upon the magnitude of the field and does not depend upon the velocity. What is the nature of the field ?
42. What is the force that a conductor $\vec{dl}$, carrying a current I experiences when placed in a magnetic field $\vec{B}$. What is the direction of the force ? [CBSE OD 90]
43. An electron beam is moving horizontally in a tube. The vertical component of earth's magnetic field is directed downwards. In which direction will the electron beam be deflected ?
44. A charged particle moves in a uniform magnetic field at right angles to the direction of the field. Which of the following quantities will change : speed, velocity, momentum, kinetic energy, displacement ?
45. In which orientation is the force experienced by a current-carrying conductor placed in a magnetic field maximum ?
46. A current carrying conductor does not tend to deflect in a magnetic field. What conclusion can be drawn from it ?
47. Name the rule that gives the direction of force on a current-carrying conductor placed perpendicular to the magnetic field.
48. Write an expression for the force between two parallel short wires carrying currents.
49. Two current elements are placed a certain distance apart but not parallel to each other. Do they exert equal and opposite forces on each other ?
50. What is the direction of force between two parallel wires carrying currents in opposite directions ?
51. The force existing between two parallel current carrying conductors is F. If the current in each conductor is doubled, what is the value of the force between them ?
52. Is the force between two parallel current-carrying wires affected by the nature of the dielectric medium between them ?
53. What is the value of net force acting on a current carrying (*i*) rectangular and (*ii*) circular loop, placed in a uniform magnetic field ? What do you expect about the torque in each case ?
54. Write an expression for the torque acting on a current carrying coil located in a uniform magnetic field.
55. Write an expression for the magnitude of the torque acting on a current carrying coil placed in a uniform radial magnetic field.
56. Under what circumstances will a current carrying loop not rotate in the magnetic field ?
57. State the principle of working of a moving coil galvanometer. [CBSE D 15]
58. What do you mean by the figure of merit of a galvanometer ?

59. Define the current sensitivity of a moving coil galvanometer and state its SI unit. [CBSE OD 13, 13C]

60. Write two factors by which the current sensitivity of a moving coil galvanometer can be increased. [CBSE D 01 ; F 08]

61. Define voltage sensitivity of a moving coil galvanometer. Give its SI unit. [Punjab 91]

62. Write two factors by which voltage sensitivity of a moving coil galvanometer can be increased. [CBSE D 01 ; F 08]

63. What is the nature of the magnetic field in a moving coil galvanometer ? [CBSE D 96 ; OD 96]

64. State two properties of the material of the wire used for suspension of the coil in a moving coil galvanometer. [CBSE OD 01, 06C]

65. The current sensitivity of a moving coil galvanometer is 5 division/mA and voltage sensitivity is 20 division/volt. Find the resistance of the galvanometer.

66. An electron and a proton, having equal momenta, enter a uniform magnetic field at right angles to the field lines. What will be the ratio of curvature of their trajectories ? [CBSE Sample Paper 05]

67. An electron is moving with a velocity v, along the axis of a long straight solenoid, carrying a current I. What will be the force acting on the electron due to the magnetic field of the solenoid ? [CBSE Sample Paper 05]

68. Among alpha, beta and gamma radiations, which get deflected by the magnetic field ? [CBSE F 04]

69. A solenoid coil of 300 turns/m is carrying a current of 5 A. The length of the solenoid is 0.5 m and has a radius of 1 cm. Find the magnitude of the magnetic field inside the solenoid. [CBSE F 04]

70. What is the resistance of an ideal ammeter ?

71. What is the resistance of an ideal voltmeter ?

72. Why should an ammeter have a high current carrying capacity ?

73. Why should a voltmeter have a low current carrying capacity ?

74. What is the effective resistance of an ammeter if a shunt of resistance R_s is used across the terminals of a galvanometer of resistance R_g ?

75. Suppose a shunt of resistance 0.01Ω is connected across a galvanometer, what can be said about the resistance of the resulting ammeter ?

76. A student wants to increase the range of an ammeter from 1 mA to 5 mA. What should be done to the shunt resistance ?

77. What is the direction of the force acting on a charged particle q, moving with a velocity $\vec{v}$ in a uniform magnetic field $\vec{B}$? [CBSE D 08]

78. Two identical charged particles moving with same speed enter a region of uniform magnetic field. If one of these enters normal to the field direction and the other enters along a direction at 30° with the field, what would be the ratio of their angular frequencies ? [CBSE Sample Paper 08]

79. An α-particle and a proton are moving in the plane of the paper in a region where there is a uniform magnetic field ($\vec{B}$) directed normal to the plane of the paper. If the two particles have equal linear momenta, what will be the ratio of the radii of their trajectories in the field ? [CBSE Sample Paper 08]

80. Why should the spring/suspension wire in a moving coil galvanometer have low torsional constant ? [CBSE OD 08]

81. The coils, in certain galvanometers, have a fixed core made of a non-magnetic metallic material. Why does the oscillating coil come to rest so quickly in such a core ? [CBSE D 08C]

82. A long straight wire carries a current I along the positive y-direction. A particle of charge $+Q$ is moving with a velocity $\vec{v}$ along the x-axis. In which direction will the particle experience a force ? [CBSE F 13]

83. Two particles A and B of masses m and $2m$ have charges q and $2q$ respectively. Both these particles moving with velocities v_1 and v_2 respectively in the same direction enter the same magnetic field B acting normally to their direction of motion. If the two forces F_A and F_B acting on them are in the ratio 1 : 2, find the ratio of their velocities. [CBSE D 11 C]

84. A beam of α particles projected along $+x$-axis, experiences a force due to magnetic field along the $+y$-axis. What is the direction of the magnetic field ? (Fig. 4.167) [CBSE OD 10]

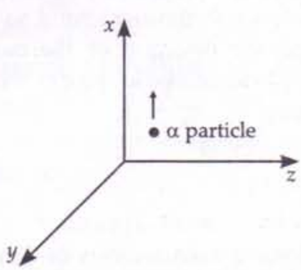


Fig. 4.167

85. A beam of electrons projected along + x-axis, experiences a force due to a magnetic field along the + y-axis. What is the direction of the magnetic field ? (Fig. 4.168) [CBSE OD 10]

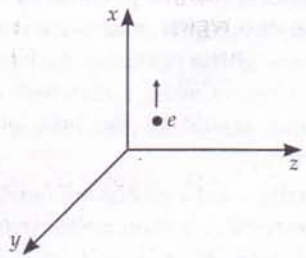


Fig. 4.168

86. A narrow stream, of protons and deuterons, having the same momentum values, enter a region of a uniform magnetic field directed perpendicular to their common direction of motion. What would be the ratio of the radii of the circular paths, described by the protons and deuterons ? [CBSE F 11]

87. A square coil, $OPQR$, of side a, carrying a current I, is placed in the Y-Z plane as shown in Fig. 4.169. Find the magnetic moment associated with this coil. [CBSE Sample Paper 13]

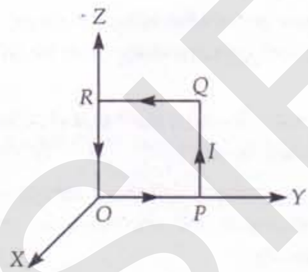


Fig. 4.169

Answers

1. A magnetic needle brought close to a straight current-carrying wire aligns itself perpendicular to the wire, reversing the direction of current reverses the direction of deflection.
2. According to Biot-Savart law, the magnetic field due to a current element $I\,\vec{dl}$ at the observation point whose position vector is $\vec{r}$ is given by
$$d\vec{B} = \frac{\mu_0 I}{4\pi}\cdot\frac{\vec{dl}\times\vec{r}}{r^3}$$
where μ_0 is the permeability of free space.
3. $K = \frac{\mu_0}{4\pi} = 10^{-7}\,\text{TmA}^{-1}$.
4. SI unit of permeability $(\mu_0) = \text{TmA}^{-1}$ or $\text{WbA}^{-1}\text{m}^{-1}$.
5. $\frac{4\pi}{\mu_0} = 10^7\,\text{T}^{-1}\text{m}^{-1}\text{A}$.
6. The direction of magnetic field due to a straight conductor can be determined with the help of *right hand thumb rule*. According to this rule if we grasp the conductor in the right hand so that the thumb points in the direction of the current, then the magnetic field will be in the direction of the curl of the fingers.
7. $B = \frac{\mu_0 I}{2\pi a}$.
8. See Fig. 4.8.
9. The magnetic field consists of concentric circular lines of force with the conductor at their centre and in a plane perpendicular to the conductor.
10. At the centre of the current loop.
11. (*i*) Magnetic field is minimum (zero) along the axis of a current element.
(*ii*) Magnetic field due to current element is maximum in a plane passing through the element and perpendicular to its axis.
12. See Fig. 4.25.
13. Wb m^{-2} is the SI unit of magnetic field B which is a vector quantity.
14. According to right hand rule, the direction of the field is (*a*) towards west above the wire and (*b*) towards east below the wire.
15. A current carrying loop behaves as a bar magnet because
(*i*) it possesses a magnetic dipole moment ($m = IA$), and
(*ii*) it experiences a torque in an external magnetic field. This torque tends to align the axis of the loop along the direction of the field.
16. See Fig. 4.25.
17. The magnetic field well inside a long solenoid having n turns per unit length and carrying current I is $B = \mu_0 nI$.
18. An anchor ring around which a large number of turns of a metallic wire are wound is called a toroid.
19. The force experienced by a charged particle while moving through a region of magnetic field is called *magnetic Lorentz force*.

It is given by $\vec{F} = q(\vec{v}\times\vec{B})$.

20. $\vec{F} = q(\vec{v} \times \vec{B})$. The direction of the force is perpendicular to the plane containing vectors $\vec{v}$ and $\vec{B}$.

21. $F = qvB \sin\theta$.

22. For a stationary charge, $v = 0$.
Therefore, $F = qvB\sin\theta = q(0)\,B\sin\theta = 0$.

23. Zero, because a magnetic force acts perpendicular to the direction of velocity or the direction of motion of the charged particle.

24. $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$.

25. Force on a charge moving along the direction of the magnetic field is zero $F = qvB\sin 0° = 0$.

26. *Fleming's left hand* rule gives the direction of force on a charged particle moving in a magnetic field : Stretch the thumb and the first two fingers of the left hand so that they are perpendicular to each other. If the forefinger points in the direction of magnetic field, central finger in the direction of current, then the thumb gives the direction of force on the charged particle.

27. Towards west.

28. Circular path.

29. The path of the particle will be a helix with its axis along the field B.

30. The charged particle will move along a straight line path.

31. The magnetic force acts perpendicular to the direction of motion of the charged particle. No work is done by the magnetic force on it. The particle does not gain any energy.

32. Magnetic force on electron = Centripetal force

$$evB\sin 90° = \frac{mv^2}{r} \qquad \therefore \text{ Radius, } r = \frac{mv}{eB}.$$

33. According to Fleming's left hand rule, the magnetic field acts in the +Z-direction.

34. Radius, $r = \frac{mv}{eB}$ i.e., $r \propto m$

As electron has smaller mass than proton, so it will circulate in a circular path of smaller radius.

35. Radius of curvature,

$$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2K}{m}} = \frac{\sqrt{2mK}}{qB} \quad i.e., r \propto \sqrt{K}$$

$\therefore$ If the kinetic energy is halved, radius of curvature is reduced to $1/\sqrt{2}$ times its initial value.

36. For undeflected beam,

$$F_m = F_e \quad \text{or} \quad evB\sin 90° = eE \quad \text{or} \quad v = \frac{E}{B}.$$

37. Zero, because the observation point lies on the axis of the straight conductor.

38. Refer answer to Q. 16 on page 4.34.

39. Time spent by a proton inside the dees of a cyclotron is independent of both its speed and radius of its circular path.

40. The field is magnetic in nature.

41. The field is electric in nature.

42. $d\vec{F} = I(\vec{dl} \times \vec{B})$

The direction of force $d\vec{F}$ is perpendicular to the plane of $\vec{dl}$ and $\vec{B}$ and will point in the same direction in which a right-handed screw, when rotated from $\vec{dl}$ to $\vec{B}$, will advance.

43. Towards west.

44. Only velocity, momentum and displacement will change as they are all vectors.

45. When the conductor is held perpendicular to the magnetic field, it experiences a maximum force.

46. This means that no force is acting on the current carrying wire due to the magnetic field. This is possible when the conductor is parallel to the direction of the magnetic field.

47. Fleming's left hand rule.

48. The force between two parallel short wires of lengths dl_1 and dl_2, separated by distance r and carrying currents I_1 and I_2 respectively, is given by

$$dF = \frac{\mu_0}{4\pi} \cdot \frac{I_1 I_2 dl_1 dl_2}{r^2}$$

49. Like other forces, these forces also obey Newton's third law of action and reaction and are, therefore, equal and opposite.

50. The direction of force is perpendicular to the two wires and is outwards, so that the two wires repel each other.

51. The value of force is $4F$. This is because force between two parallel current carrying conductors is proportional to the product of the currents through them.

52. No. This interaction is between the magnetic fields produced by the two wires which does not depend on the nature of the dielectric medium.

53. In each case the net force is zero but torque is non-zero.

54. If a coil of area A, turns N and carrying current I is held in a uniform magnetic field B, it experiences a torque given by

$\tau = NIBA\sin\theta$, where θ is the angle between $\vec{B}$ and the normal to the plane of the loop.

55. $\tau = NIBA$.

56. If the current carrying loop is placed in a magnetic field, with its plane perpendicular to the field, then it will not rotate.

57. Refer to point 26 of Glimpses.

58. The figure of merit of a galvanometer is defined as the amount of current required to produce one scale deflection in the galvanometer.

It is given by $G = \frac{I}{\alpha} = \frac{k}{NBA}$

59. The current sensitivity of a galvanometer is defined as the deflection produced in the galvanometer on passing unit current through it.

Current sensitivity $= \frac{\alpha}{I} = \frac{NBA}{k}$

The SI unit of current sensitivity is **radian ampere^{-1}**.

60. The current sensitivity of a moving coil galvanometer can be increased by (*i*) increasing the number of turns in the galvanometer coil. (*ii*) decreasing the torsion constant of its suspension fibre.

61. The voltage sensitivity of a moving coil galvanometer is defined as the deflection produced in the galvanometer when a unit voltage is applied across its coil.

Voltage sensitivity $= \frac{\alpha}{V} = \frac{\alpha}{IR} = \frac{NBA}{kR}$

The SI unit of voltage sensitivity is **radian volt^{-1}**.

62. The voltage sensitivity of a moving coil galvanometer can be increased by

(*i*) increasing the number of turns of the galvanometer coil

(*ii*) decreasing the torsion constant of the suspension fibre.

63. Radial magnetic field is used in a moving coil galvanometer.

64. The material used for the suspension wire of a moving coil galvanometer should have following properties :

(*i*) Small torsion constant k which makes the galvanometer highly sensitive.

(*ii*) High tensile strength so that even thin wire does not break under the weight of the suspension coil.

65. Here $I_S = 5\text{ div mA}^{-1} = 5 \times 10^3 \text{div A}^{-1}$,

$$V_S = 20 \text{ div V}^{-1}$$

$$R_g = \frac{I_S}{V_S} = \frac{5 \times 10^3}{20} = \mathbf{250\,\Omega}.$$

66. As $r = \frac{mv}{eB}$ *i.e.*, $r \propto mv$ $\therefore$ $r_e : r_p = \mathbf{1:1}$

67. $F = evB \sin 0° = \mathbf{0}$.

68. Alpha and beta radiations are deflected by the magnetic field.

69. $B = \mu_0 nI = 4\pi \times 10^{-7} \times 300 \times 5 = \mathbf{1.9 \times 10^{-3}\ T}$.

70. Zero.

71. Infinite.

72. Due to high current carrying capacity, an ammeter is not damaged by excessive currents.

73. Due to low current carrying capacity, the voltmeter will draw only a small part of the total current. The potential difference ($V = IR$) to be measured will not be much different from the actual value.

74. Effective resistance of ammeter

$$R_A = \frac{R_s R_g}{R_s + R_g}.$$

75. The resistance of the resulting ammeter will be less than $0.01\,\Omega$.

76. The value of shunt resistance should be reduced so that more current may pass through it.

77. The force $\vec{F}$ acts in the direction of the vector $\vec{v} \times \vec{B}$ *i.e.*, perpendicular to the plane of the vectors $\vec{v}$ and $\vec{B}$.

78. Angular frequency, $\omega = \frac{qB}{m}$. It is independent of angle θ.

$\therefore$ Ratio of the angular frequencies, $\omega_1 : \omega_2 = \mathbf{1:1}$.

79. Radius, $r = \frac{mv}{qB} = \frac{p}{qB}$

For same p and B, $\frac{r_\alpha}{r_p} = \frac{q_p}{q_\alpha} = \frac{e}{2e} = \frac{1}{2} = \mathbf{1:2}$

80. Low torsional constant of the suspension wire ensures high sensitivity of the moving coil galvanometer.

81. The eddy currents set up in the metallic material oppose the motion of the coil in the magnetic field and hence bring it to rest at once.

82. The field $\vec{B}$ due to current I acts along –ve z-direction. By Fleming's left hand rule, the charge $+Q$ will experience a force along +ve y-direction.

83. $\frac{F_A}{F_B} = \frac{qv_1 B \sin 90°}{2qv_2 B \sin 90°} = \frac{1}{2}$ $\therefore$ $\frac{v_1}{v_2} = \mathbf{1:1}$.

84. By Fleming's left hand rule, the magnetic field acts along $\pm$ z-axis.

85. By Fleming's left hand rule, the magnetic field is directed along + z-axis.

86. Radius, $r = \frac{mv}{qB} = \frac{p}{qB}$

For same p and B, $\frac{r_p}{r_d} = \frac{q_d}{q_p} = \frac{e}{e} = \mathbf{1 : 1}$.

87. By right hand rule, the direction of magnetic moment will be along +ve X-direction.

$\therefore \quad \vec{m} = IA(+\hat{i}) = Ia^2\,\hat{i}$

TYPE B : SHORT ANSWER QUESTIONS (2 or 3 marks each)

1. Briefly describe Oersted's experiment leading to the discovery of magnetic effect of current. State Ampere's swimming rule.
2. State Biot-Savart's law in vector form expressing the magnetic field due to an element $\vec{dl}$ carrying current I at a distance $\vec{r}$ from the element. How will you find the direction of the magnetic field ? [ISCE 93 ; CBSE D 02 C ; OD 14C]
3. A wire of length L is bent into a semi-circular loop. Use Biot-Savart law to deduce an expression for the magneitc field at the centre due to current I passing through it. [CBSE D 11 C]
4. Using Biot-Savart's law, derive the expression for the magnitude of the magnetic field at the centre of a circular loop of radius r carrying a steady current I. Draw the field lines due to the current loop. [ISCE 96 ; CBSE D 01C ; OD 14 C]
5. State Biot-Savart law. Deduce the expression for the magnetic field at a point on the axis of a current carrying circular loop of radius 'R', distant 'x' from the centre. Hence write the magnetic field at the centre of a loop. [CBSE D 05 ; OD 05, 15]
6. State Ampere's circuital law and prove this law for a circular path around a long current carrying conductor. [Himachal 98 ; Haryana 98C, 01]
7. Using Ampere's circuital theorem, calculate the magnetic field due to an infinitely long wire carrying current I. [CBSE OD 90]
8. A long solenoid with closely wound turns has n turns, per unit of its length. A steady current I flows through this solenoid. Use Ampere's circuital law to obtain an expression, for the magnetic field, at a point on its axis and close to its mid point. Draw its field lines. [CBSE D 04 C, 14C]
9. (*a*) How is a toroid different from a solenoid ?
   (*b*) Use Ampere's circuital law to obtain the magnetic field inside a toroid.
   (*c*) Show that in an ideal toroid, the magnetic field (*i*) inside the toroid and (*ii*) outside the toroid at any point in the open space is zero. [CBSE OD 08, 14C]
10. (*a*) State Ampere's circuital law, expressing it in the integral form.
    (*b*) Two long coaxial insulated solenoids, S_1 and S_2 of equal lengths are wound one over the other as shown in the figure. A steady current "I" flows through the inner solenoid S_1 to the other end B, which is connected to the outer solenoid S_2 through which the same current "I" flows in the opposite direction so as to come out at end A. If n_1 and n_2 are the number of turns per unit length, find the magnitude and direction of the net magnetic field at a point (*i*) inside on the axis and (*ii*) outside the combined system. [CBSE D 14]

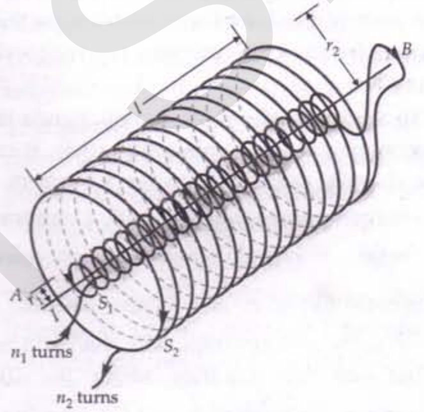


Fig. 4.170

11. A long straight wire of a circular cross-section of radius 'a' carries a steady current 'I'. The current is uniformly distributed across the cross-section. Apply Ampere's circuital law to calculate the magnetic field at a point 'r' in the region for (*i*) $r < a$ and (*ii*) $r > a$. [CBSE D 10]
12. Write an expression for force $\vec{F}$ acting on a charge q moving with a velocity $\vec{v}$ in the region, where magnetic induction $\vec{B}$ is uniform. How does the speed change, as the charge moves ? Under what circumstances the force $\vec{F}$ shall be zero ? [ISCE 96]
13. Write an expression for the force on a charge moving in a magnetic field. Use this expression to define the SI unit of magnetic field. [CBSE D 08C]
14. Consider the motion of a charged particle of mass 'm' and charge 'q' moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$.

(*a*) If $\vec{v}$ is perpendicular to $\vec{B}$, show that it describes a circular path having angular frequency $\omega = qB/m$.

(*b*) If the velocity $\vec{v}$ has a component parallel to the magnetic field $\vec{B}$, trace the path described by the particle. Justify your answer. [CBSE D 14C]

15. A uniform magnetic field $\vec{B}$ is set up along the positive *x*-axis. A particle of charge '*q*' and mass '*m*' moving with a velocity $\vec{v}$ enters the field at the origin in X - Y plane such that it has velocity components both along and perpendicular to the magnetic field $\vec{B}$. Trace, giving reason, the trajectory followed by the particle. Find out the expression for the distance moved by the particle along the magnetic field in one rotation. [CBSE OD 15]

16. A steady magnetic field cannot change the kinetic energy of a moving charged particle, it can deflect the charged particle sideways. Comment.

17. A charged particle moving with a uniform velocity $\vec{v}$ enters a region where uniform electric and magnetic fields $\vec{E}$ and $\vec{B}$ are present. It passes through the region without any change in its velocity. What can we conclude about the (*i*) relative directions of $\vec{B}$, $\vec{v}$, and $\vec{B}$? (*ii*) magnitudes of $\vec{E}$ and $\vec{B}$. [CBSE Sample Paper 08]

18. A hydrogen ion of mass '*m*' and charge '*q*' travels with a speed '*v*' along a circle of radius '*r*' in a uniform magnetic field of flux density '*B*'. Obtain the expression for the magnetic force on the ion and determine its time period. [CBSE D 03C ; OD 04]

19. Show that the frequency of revolution, of a charged particle (in the *X*-*Y* plane), in a uniform magnetic field $\vec{B}$ ($\vec{B} = B\hat{k}$), is independent of its speed.

Which practical machine makes use of this fact ? What is the frequency of the alternating electric field, used in this machine ? [CBSE D 09C]

20. Draw a schematic sketch of a cyclotron. Explain clearly the role of crossed electric and magnetic fields in accelerating the charge. Hence derive the expression for the kinetic energy acquired by the particles. [CBSE OD 13]

21. Derive an expression for the maximum force experienced by a straight conductor of length *l*, carrying current *I* and kept in a uniform magnetic field, *B*. [CBSE D 06C]

22. Briefly describe an experiment to show the existence of a repulsive force between two parallel conductors carrying currents in opposite directions.

23. Derive the expression for force per unit length between two long straight parallel current carrying conductors. Hence define one ampere. [CBSE D 01, 09]

24. Derive a formula for the force between two parallel straight conductors carrying current in opposite directions and write the nature of the force. Hence, define an ampere. [CBSE OD 98]

25. Derive an expression for the torque on a rectangular coil of area *A*, carrying a current *I* and placed in a magnetic field *B*. The angle between the direction of *B* and vector perpendicular to the plane of the coil is θ. Indicate the direction of the torque acting on the loop. [CBSE F 09]

26. A rectangular coil of sides '*l*' and '*b*' carrying a current *I* is subjected to a uniform magnetic field $\vec{B}$ acting perpendicular to its plane. Obtain the expression for the torque acting on it. [CBSE D 14C]

27. A rectangular loop of area *A*, having *N* turns and carrying a current of *I* ampere is held in a uniform magnetic field *B*. (*i*) Write the expression for the maximum torque experienced by the loop. (*ii*) In which orientation, will the loop be in stable equilibrium ? [CBSE OD 98C]

28. State the principle of a moving coil galvanometer. Show that the current passing through the coil is directly proportional to the deflection of the coil. [Haryana 02]

29. A moving coil galvanometer consists of a rectangular coil of *N* turns, each of area *A*, suspended in a radial magnetic field of flux density *B*. Derive the expression for the torque on the coil, when current *I* passes through it. Draw suitable labelled diagram. [CBSE D 93C]

30. A moving coil galvanometer of resistance *G* gives a full scale deflection for a current I_g. Use the suitable circuit diagram to convert it into an ammeter of range 0 to I ($I > I_g$). Deduce the expression for the shunt requried for this conversion. Hence write the expression for the resistance of the ammeter thus obtained. [Punjab 2000 ; CBSE D 09C]

31. Explain how will you convert a galvanometer into a voltmeter to read a maximum potential difference of *V* volts. Can one use a voltmeter to measure the emf of a cell ? Justify your answer. [CBSE OD 97C, F 98]

Answers

1. Refer answer to Q. 2 on page 4.1.
2. Refer answer to Q. 3 on page 4.2.
3. Refer to solution of Example 17 on page 4.16.
   Here $L = \pi r$ or $r = L/\pi$

$$B = \frac{\mu_0 I}{4r}$$

$$= \frac{\mu_0 I}{4} \cdot \frac{\pi}{L} = \frac{\pi \mu_0 I}{4L}.$$

4. Refer answer to Q. 7 on page 4.12. See Fig. 4.25.
5. Refer answer to Q. 8 on page 4.13. See Fig. 4.25 on page 4.14.
6. Refer answer to Q. 9 on page 4.22.
7. Refer to solution of Example 33 on page 4.26.
8. Refer answer to Q. 10 on page 4.23.
9. Refer answer to Q. 11 on page 4.24.
10. (*a*) $\oint \vec{B} \cdot \vec{dl} = \mu_0 I$

    (*b*) (*i*) Magnetic field inside solenoid S_1,

$$B_1 = \mu_0 n_1 I$$

Magnetic field inside solenoid S_2,

$$B_2 = \mu_0 n_2 I$$

As the currents in the two solenoids are oppositely directed, so direction of B_2 is opposite to that of B_1. The net magnetic field at any inside point along the axis,

$$B = B_1 - B_2 = \mu_0 (n_1 - n_2) I$$

(*ii*) Outside the combined system, net magnetic field = 0.

11. Refer to the solution of Example 33(*i*) on page 4.26.
12. Force, $\vec{F} = q(\vec{v} \times \vec{B})$. The speed of the charge is not affected in the magnetic field. Force $\vec{F}$ will be zero if $\vec{v} = 0$ or if $\vec{v}$ is parallel or antiparallel to $\vec{B}$.
13. Refer answer to Q. 12 on page 4.28.
14. (*a*) Magnetic force acts on the charged particle in a direction perpendicular to both $\vec{v}$ and $\vec{B}$ and provides centripetal force.

$\therefore$ Magnetic force, $qvB \sin 90°$

$$= \text{Centripetal force, } \frac{mv^2}{r} \quad \text{or} \quad r = \frac{mv}{qB}$$

$$\therefore \quad \omega = \frac{v}{r} = \frac{qB}{m}$$

(*b*) Refer answer to Q. 15(3) on page 4.34.

15. Refer answer to Q. 15(3) on page 4.34
16. A magnetic field exerts force in a direction perpendicular to the direction of motion of the charge. No work is done by the magnetic force on the moving charge. So the kinetic energy of the charged particle is not affected. The perpendicular magnetic force only deflects the charged particle sideways.
17. (*i*) The directions of $\vec{E}$ and $\vec{B}$ are perpendicular to each other and also perpendicular to the direction of $\vec{v}$ so that electric and magnetic forces are in opposite directions.

    (*ii*) The magnitudes of $\vec{E}$ and $\vec{B}$ should be such that

$$qE = qvB$$

or
$$v = \frac{E}{B}.$$

18. In the uniform magnetic field, the magnetic force on the hydrogen ion acts perpendicular to both v and B.

$$\therefore \quad F = evB \sin 90° = evB$$

Magnetic force on the hydrogen ion = Centripetal force

or
$$evB = \frac{mv^2}{r}$$

or
$$r = \frac{mv}{eB}$$

Time period of hydrogen ion,

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v} \cdot \frac{mv}{eB} = \frac{2\pi m}{eB}.$$

19. Refer answer to Q. 15(2) on page 4.33.

We obtain, $f_c = \dfrac{qB}{2\pi m}$

A cyclotron makes use of this fact in which alternating electric field of frequency f_c is applied.

20. Refer answer to Q. 17 on page 4.40.
21. Refer answer to Q. 19 on page 4.44.
22. Refer answer to Q. 20 (Experiment 2) on page 4.49.
23. Refer answer to Q. 21 on page 4.49.
24. Refer answer to Q. 21 on page 4.49.
25. Refer answer to Q. 22 on page 4.53.
26. Magnetic moment associated with the current carrying coil is

$$\vec{m} = IA\hat{n} = Ilb\hat{n}$$

where $\hat{n}$ is a unit vector perpendicular to the plane of the coil.

Torque, $\vec{\tau} = \vec{m} \times \vec{B} = Ilb\,\hat{n} \times \vec{B} = Ilb\,\vec{0} = \vec{0}$

This is because $\hat{n}$ and $\vec{B}$ are either parallel or antiparallel vectors.

27. Refer answer to Q. 22 on page 4.53. The loop will be in stable equilibrium when $\vec{m}$ is parallel to $\vec{B}$.

28. Refer answer to Q. 23 on page 4.57.

29. Refer answer to Q. 23 on page 4.57.

30. Refer answer to Q. 26 on page 4.63.

31. Refer answer to Q. 28 on page 4.64. No, a voltmeter cannot be used to measure the emf a cell. A voltmeter requires a small current for its operation. It measures p.d. in a closed circuit, which is less than the emf of the cell.

TYPE C : LONG ANSWER QUESTIONS (5 marks each)

1. (*a*) Write any two important points of similarities and differences each between Coulomb's law for the electrostatic field and Biot-Savart's law for the magnetic field.
   (*b*) Use Biot-Savart's law to find the expression for the magnetic field due to a circular loop of radius 'r' carrying current 'I', at its centre. [CBSE F 15]

2. State Biot-Savart law expressing it in vector form. Use it to obtain the magnetic field, at an axial point, distant r from the centre of a circular coil of radius a carrying a current I. Hence, compare the magnitudes of the magnetic field of this coil at the centre and at an axial point for which $r = \sqrt{3}\,a$. [CBSE SP 08]

3. (*a*) Use Biot-Savart law to derive the expression for the magnetic field due to a circular coil of radius R having N turns at a point on the axis at a distance 'x' from its centre.
   Draw the magnetic field lines due to this coil.
   (*b*) A current 'I' enters a uniform circular loop of radius 'R' at point M and flows out at N as shown in the figure.
   Obtain the net magnetic field at the centre of the loop. [CBSE D 15C]

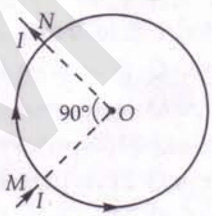


Fig. 4.171

4. (*a*) Write, using Biot-Savart law, the expression for the magnetic field element $\vec{dl}$ carrying current I at a distance $\vec{r}$ from it in a vector form.
   Hence derive the expression for the magnetic field due to a current carrying loop of radius R at a point P distant x from its centre along the axis of the loop.
   (*b*) Explain how Biot-Savart law enables one to express the Ampere's circuital law in the integral form, *viz.*,
   $$\oint \vec{B} \cdot \vec{dl} = \mu_0 I,$$
   where I is the total current passing through the surface. [CBSE OD 15]

5. (*a*) State Ampere's circuital law connecting the line integral of $\vec{B}$ over a closed path to the net current crossing the area bounded by the path.
   (*b*) Use Ampere's law to derive the formula for the magnetic field due to an infinitely long straight current carrying wire.
   (*c*) Explain carefully why the derivation as in (*b*) is not valid for magnetic field in a plane normal to a current-carrying straight wire of finite length and passing through the midpoint of the axis.

6. (*a*) Show how Biot-Savart law can be alternatively expressed in the form of Ampere's circuital law. Use this law to obtain the expression for the magnetic field inside a solenoid of length 'l', cross-sectional area 'A' having 'N' closely wound turns and carrying a steady current 'I'.
   (*b*) Sketch the magnetic field lines for a finite solenoid. Explain why the field at the exterior midpoint is weak while at the interior it is uniform and strong. [CBSE D 06C, 15C]

7. (*a*) State Ampere's circuital law. Use this law to obtain the expression for the magnetic field inside an air cored toroid of average radius 'r', having 'n' turns per unit length and carrying a steady current I. Show that the magnetic field in the open space inside and exterior to the toroid is zero.

(b) An observer to the left of a solenoid of N turns each of cross-section area 'A' observes that a steady current I in it flows in the clockwise direction. Depict the magnetic field lines due to the solenoid specifying its polarity and show that it acts as a bar magnet of magnetic moment $m = NIA$. [CBSE OD 13, D 15]

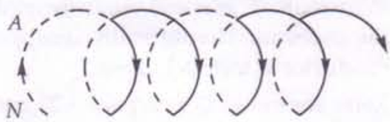


Fig. 4.172

8. (a) Using Ampere's circuital law, obtain the expression for the magnetic field due to a long solenoid at a point inside the solenoid on its axis.
   (b) In what respect is a toroid different from a solenoid ? Draw and compare the pattern of the magnetic field lines in two cases.
   (c) How is the magnetic field inside a given solenoid made strong ? [CBSE OD 11]
9. Derive a mathematical expression for the force acting on a current carrying straight conductor kept in a magnetic field. State the rule used to determine the direction of this force. Under what conditions is this force (i) zero and (ii) maximum ? [CBSE D 97C, 98]
10. Draw a schematic sketch of a cyclotron. Explain briefly how it works and how it is used to accelerate the charged particles.
    (i) Show that time period of ions in a cyclotron is independent of both the speed and radius of circular path.
    (ii) What is resonance condition ? How is it used to accelerate the charged particles ? [CBSE D 08 ; OD 09]
11. With the help of a labelled diagram, state the underlying principle of a cyclotron. Explain clearly how it works to accelerate the charged particles.
    Show that cyclotron frequency is independent of energy of the particle. Is there an upper limit on the energy acquired by the particle ? Give reason. [CBSE D11, 14, 14C]
12. (a) Derive an expression for the force between two long parallel current carrying conductors.
    (b) Use this expression to define SI unit of current.
    (c) A long straight wire AB carries a current I. A proton P travels with a speed v, parallel to the wire, at a distance d from it in a direction opposite to the current as shown in Fig. 4.173. What is the force experienced by the proton and what is its direction ? [CBSE D 06 ; OD 10]

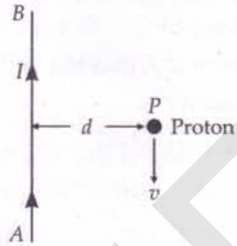


Fig. 4.173

13. Derive an expression for the torque acting on a loop of N turns, area A, carrying current I, when held in a magnetic field B. With the help of a circuit diagram, show how a moving coil galvanometer can be converted into an ammeter of given range. Write the necessary mathematical formula. [CBSE D 04]
14. (a) Two straight long parallel conductors carry currents I_1 and I_2 in the same direction. Deduce the expression for the force per unit length between them.
    Depict the pattern of magnetic field lines around them.
    (b) A rectangular current carrying loop $EFGH$ is kept in a uniform magnetic field as shown in Fig. 4.174.

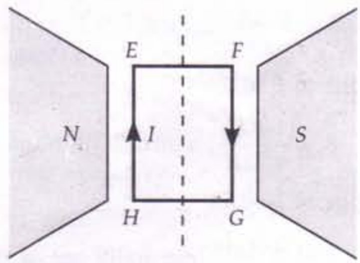


Fig. 4.174

    (i) What is the direction of the magnetic moment of the current loop ?
    (ii) When is the torque acting on the loop (A) maximum, (B) zero ? [CBSE OD 05, 09]
15. (a) With the help of a diagram, explain the principle and working of a moving coil galvanometer.
    (b) What is the importance of a radial magnetic field and how is it produced ?
    (c) Why is it necessary to introduce a cylindrical soft iron core inside the coil of a galvanometer ?
    (d) "Increasing the current sensitivity of a galvanometer may not necessarily increase its voltage sensitivity". Justify this statement. [CBSE D 06, 13C ; OD 14, 14C, 15]

Answers

1. (*a*) Refer answer to Q. 4 on page 4.3.
   (*b*) Refer answer to Q. 7 on page 4.12.
2. Refer answer to Q. 8 on page 4.13.

$$B_{axial} = \frac{\mu_0 \, I a^2}{2(r^2 + a^2)^{3/2}}$$

$$B_{r=\sqrt{3}a} = \frac{\mu_0 \, I a^2}{2(3a^2 + a^2)^{3/2}} = \frac{\mu_0 \, I}{16a}$$

$$B_{centre} = \frac{\mu_0 \, I a^2}{2(0^2 + a^2)^{3/2}} = \frac{\mu_0 \, I}{2a} \qquad [\text{Put } r = 0]$$

$$\therefore \frac{B_{centre}}{B_{r=\sqrt{3}a}} = \frac{16a}{2a} = 8.$$

3. (*a*) Refer answer to Q. 8 on page 4.13. See Fig. 4.25.
   (*b*) At point *M*, let the current *I* be divided into two parts : I_1 along the smaller part and I_2 along the larger part of the loop.

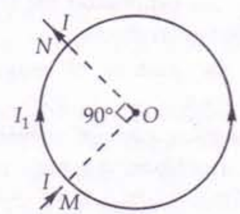


Fig. 4.175

Field due to I_1 at *O*,

$$\vec{B}_1 = \frac{1}{4} \cdot \frac{\mu_0 I_1}{2R}, \text{ normally into the paper.}$$

Field due to I_2 at *O*,

$$\vec{B}_2 = \frac{3}{4} \cdot \frac{\mu_0 I_2}{2R}, \text{ normally out of the paper}$$

Net field at *O*, $\vec{B} = \vec{B}_1 + \vec{B}_2$

$$|\vec{B}| = \frac{1}{4}\frac{\mu_0 I_1}{2R} - \frac{3}{4}\frac{\mu_0 I_2}{2R}$$

As the resistance of the larger part is 3 times the resistance of the smaller part, so $I_1 = 3I_2$

Hence, $|\vec{B}| = 0$.

4. (*a*) Refer answer to Q. 8 on page 4.13.
   (*b*) Biot-Savart law can be expressed as Ampere's circuital law by considering the surface to be made up of a large number of loops. The sum of the tangential components of the magnetic field multiplied by the length of all such elements, gives the result $\oint \vec{B} \cdot \vec{dl} = \mu_0 I$.
5. (*a*) Refer answer to Q. 9 on page 4.22.
   (*b*) Refer to solution of Example 33 on page 4.26.
   (*c*) A straight conductor of finite length cannot by itself form a complete steady current circuit. Additional conductors are necessary to close the circuit. These will spoil the symmetry of the problem. The difficulty disappears if the conductor is infinitely long.
6. (*a*) Refer answer to Q.9 on page 4.22 and Q. 10 on page 4.23.
   (*b*) The magnetic field due to the neighbouring turns add up along the axis of the solenoid and tend to cancel out in the perpendicular direction. Thus the field at the exterior mid-point is weak and at the interior, it is uniform and strong.
7. (*a*) Refer answer to Q. 9. on page 4.22 and Q. 11 on page 4.24.
   (*b*)

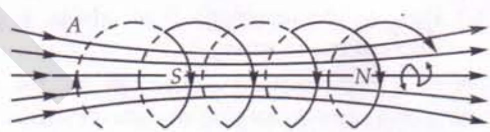


Fig. 4.176

The solenoid consists of *N* loops, each of area *A* and carrying a current *I*. Each loop acts as a magnetic dipole of dipole moment $m = IA$. As the magnetic moments of all loops are aligned along the same direction, so the net magnetic moment of the solenoid is *NIA*.

8. (*a*) Refer answer to Q. 10 on page 4.23.
   (*b*) A solenoid bent into the form of a closed ring is called a toroidal solenoid. The field pattern of solenoid is similar to that of a bar magnet. The field lines inside a toroid are circular loops and the field is uniform everywhere inside the toroid. See Fig. 4.48 on page 4.23 and Fig. 4.52 on page 4.24.
   (*c*) The field inside a solenoid can be increased by
   (*i*) inserting an iron core inside it
   (*ii*) increasing number of turns per unit length, and
   (*iii*) increasing the current through the solenoid.
9. Refer answer to Q. 19 on page 4.44.
10. Refer answer to Q. 17 on page 4.40.
11. For cyclotron, refer answer to Q. 17 on page 4.40.
    Cyclotron frequency, $f_c = \frac{qB}{2\pi m}$

As f_c is independent of velocity v, so f_c is independent of the kinetic energy of the particle.

According to Einstein's special theory of relativity, the mass of a particle increases with its velocity. At high velocities the cyclotron frequency will decrease due to increase in mass. This will throw the particle out of resonance with the oscillatory field. Hence the particles are not accelerated further.

12. (*a*), (*b*) Refer answer to Q. 21 on page 4.49.

(*c*) The field due to current I at point P is,

$$B = \frac{\mu_0 I}{2\pi d}$$

This field acts normally into the plane of paper. According to Fleming's left hand rule, a force acts on the proton in a direction away from wire AB.

$$F = evB\sin 90^\circ = ev\frac{\mu_0 I}{2\pi d} = \frac{\mu_0 Iev}{2\pi d}.$$

13. Refer answer to Q. 22 on page 4.53.

14. (*a*) Refer answer to Q. 21 on page 4.49.

(*b*) (*i*) According to right hand thumb rule, the direction of the magnetic moment of the current loop will be normally into the plane of the paper.

(*ii*) Torque acting on the loop is *maximum* when its plane is *parallel* to the magnetic field. Torque acting on the loop is *zero* when its plane is *perpendicular* to the magnetic field.

15. (*a*) Refer answer to Q. 23 on page 4.57.

(*b*) Refer to the solution of Problem 29 on page 4.85.

(*c*) A soft iron core makes the field radial. It also increases the strength of the magnetic field and hence increases the sensitivity of the galvanometer.

(*d*) Refer to the solution of Problem 33 on page 4.86.

TYPE D : VALUE BASED QUESTIONS (4 marks each)

1. Dimpi's class was shown a video on effects of magnetic field on a current carrying straight conductor. She noticed that the force on the straight current carrying conductor becomes zero when it is oriented parallel to the magnetic field and this force becomes maximum when it is perpendicular to the field. She shared this interesting information with her grandfather in the evening. The grandfather could immediately relate it to something similar in real life situations. He explained it to Dimpi that similar things happen in real life too. When we align and orient our thinking and actions in an adaptive and accommodating way, our lives become more peaceful and happy. However, when we adopt an unaccommodating and stubborn attitude, life becomes troubled and miserable. We should therefore always be careful in our response to different situations in life and avoid unnecessary conflicts. [CBSE Sample Paper 15]

Answer the following questions based on above information :

(*a*) Express the force acting on a straight current carrying conductor kept in a magnetic field in vector form. State the rule used to find the direction of this force.

(*b*) Which one value is displayed and conveyed by

(*i*) grandfather as well as

(*ii*) Dimpi ?

(*c*) Mention one specific situation from your own life which reflects similar values shown by you towards your elders.

2. Deepak was performing an experiment on potentiometer in his practical period. Unfortunately, a galvanometer fell from his hands and broke. He was sad and his friend advised him not to tell the teacher about that incident. But Deepak went to his teacher and narrated the incident. The teacher heard him patiently and on finding that it was not a Deepak's fault but just an accident, did not scold him. Instead, he used the broken galvanometer to explain its internal construction to the entire class.

Based on the above paragraph, answer the following :

(*a*) What were the values displayed by Deepak ?

(*b*) State the basic principle of a moving coil galvanometer.

3. Kamal's uncle was advised by his doctor to undergo an MRI scan test of his chest and gave him an estimate of the cost. Not knowing much about the significance of this test and finding it to be too expensive he first hesitated. When Kamal learnt about this, he decided to take help of his family, friends and neighbours and arranged for the cost. He convinced his uncle to undergo this test so as to enable the doctor to diagnose the disease. He got the test done and the resulting information greatly helped the doctor to give him proper treatment. [CBSE F 13]

Based on the above paragraph, answer the following :

(a) What according to you, are the values displayed by Kamal and her family, friends and neighbours ?

(b) Assuming that the MRI scan of her uncle's chest was done by using a magnetic field of 0.1 T, find the maximum and minimum values of force that this magnetic field could exert on a proton (charge $= 1.6 \times 10^{-19}$ C) moving with a speed of 10^4 m/s. State the condition under which the force can be minimum.

Answers

1. (a) $\vec{F} = I(\vec{l} \times \vec{B})$

The direction of the force is given by Fleming's left hand rule. For statement, refer to point 11 of Glimpses on page 4.118.

(b) (i) Adaptation to different situations and flexible and adjustable attitude. (ii) Sharing excitement in classroom learning with family members.

(c) Avoiding unnecessary arguments in conflicting situations in everyday life.

2. (a) Courage to tell the truth and gratitude to the teacher for his patience and tolerance.

(b) The basic principle of moving coil galvanometer is that a current-carrying coil placed in a magnetic field experiences a torque, the magnitude of which depends on the strength of current.

3. (a) (i) Presence of mind ;

High degree of general awareness ;

Ability to take prompt decisions ;

Concern for her uncle ;

(ii) Empathy ;

Helping and caring nature.

(b) Maximum force

$$= qvB = 1.6 \times 10^{-19} \times 10^4 \times 0.1$$

$$= \mathbf{1.6 \times 10^{-16}\ N}$$

Force is maximum when $\vec{v} \perp \vec{B}$.

Minimum force = 0

Force is minimum when $\vec{v} \parallel \vec{B}$.

COMPETITION SECTION

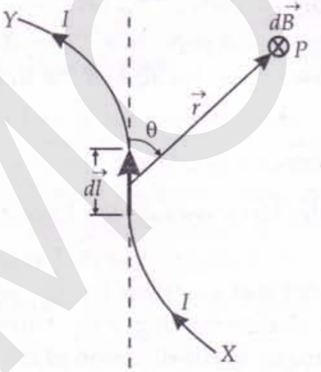
Magnetic Effect of Current

GLIMPSES

1. Oersted observation. A compass needle suffers a deflection when it is placed near a wire carrying an electric current. When the direction of current is reversed, the direction of deflection of the needle also reverses. This conclusively proves that a current carrying conductor produces a magnetic field around it. This is called *magnetic effect of current.*

2. Biot-Savart law. According to this law, the magnetic field due to a current carrying element $\vec{dl}$ carrying current I at a point P at distance r from it is given by

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I\, dl \sin\theta}{r^2}$$



In vector notation, $\vec{dB} = \frac{\mu_0 I}{4\pi} \cdot \frac{\vec{dl} \times \vec{r}}{r^3}$

Here θ is the angle between $\vec{dl}$ and $\vec{r}$ and $\mu_0 = 4\pi \times 10^{-7}$ Tm A^{-1}, is the permeability of free space. The direction of $\vec{dB}$ is same as that of $\vec{dl} \times \vec{r}$ and is given by right-hand screw rule.

3. Magnetic field due to straight current carrying conductor. The magnetic field at a point at perpendicular distance 'a' from a straight conductor carrying current I is given by

$$B = \frac{\mu_0 I}{4\pi a} (\sin\phi_1 + \sin\phi_2)$$

where ϕ_1 and ϕ_2 are the angles which the perpendicular from the observation point to the conductor makes with the lines joining the ends of the conductor to the observation point.

The magnetic field due to a straight conductor of infinite length $\left(\phi_1 = \phi_2 = \frac{\pi}{2}\right)$ is given by

$$B = \frac{\mu_0 I}{2\pi a}$$

4. Rules for finding the direction of magnetic field due to straight current carrying conductor.

   (*i*) ***Right hand thumb rule.*** If we hold the straight conductor in the grip of our right hand in such a way that the extended thumb points in the direction of current, then the direction of curl of fingers will give the direction of the magnetic field.

   (*ii*) ***Maxwell's cork screw rule.*** If a right handed screw be rotated along a wire so that it advances in the direction of current, then the direction in which the thumb rotates gives the direction of the magnetic field.

5. Magnetic field of a circular current loop. The magnetic field of a circular current loop of radius a carrying current I is

(*i*) *At the centre of the loop* : $B = \frac{\mu_0 I}{2a}$

(*ii*) *At an axial point at distance r from the centre* :

$$B = \frac{\mu_0 I a^2}{2(r^2 + a^2)^{3/2}}.$$

6. Rules for finding the direction of magnetic field due to circular current loop.

(*i*) ***Right hand thumb rule.*** If we curl the fingers of our right hand around the circular wire with the fingers pointing in the direction of the current, then the extended thumb gives the direction of the magnetic field.

(*ii*) ***Clock rule.*** This rule gives the polarity of any face of the circular current loop. If the current round any face of the coil is in anticlockwise direction, it behaves like a north pole. If the current flows in the clockwise direction, it behaves like a south pole.

7. Ampere's circuital law. The line integral of the magnetic field $\vec{B}$ around any closed path is equal to μ_0 times the total current I threading the closed path.

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$$

In a simplified form, the law states that if field $\vec{B}$ is directed along the tangent to every point on the perimeter L of a closed curve and its magnitude is constant along the curve, then

$$BL = \mu_0 I.$$

8. Magnetic field of a straight solenoid. A solenoid is a long insulated wire wound in the form of a helix. Its length is very large as compared to its diameter. The magnetic field of a straight solenoid carrying current I and having n turns per unit length is given by

(*i*) $B = \mu_0 n I$

(At a point well inside the solenoid)

(*ii*) $B_{end} = \frac{1}{2} \mu_0 n I$

(At either end of the solenoid)

9. Magnetic field of a toroidal solenoid. A solenoid bent into the form of a closed ring is called a toroidal solenoid. The magnetic field inside the toroidal solenoid has a constant magnitude and tangential direction. It is given by

$$B = \mu_0 n I$$

where I is the current in the windings and n is the number of turns per unit length. The field lines are concentric circles.

10. Force on a charge moving in a magnetic field. A charge q moving with velocity $\vec{v}$ at an angle θ with the magnetic field $\vec{B}$ experiences the *magnetic Lorentz force,*

$$F = qvB \sin\theta$$

In vector notation, $\vec{F} = q(\vec{v} \times \vec{B})$

The direction of this force is perpendicular to both $\vec{v}$ and $\vec{B}$, and work done by it is zero. This force is maximum when the charged particle moves perpendicular to the direction of the field ($\theta = 90°$) and minimum when the charged particle moves along the field ($\theta = 0°$).

11. Rules for finding the direction of force on a charge moving perpendicular to a magnetic field.

(*i*) ***Fleming's left hand rule.*** Stretch the thumb and the first two fingers of the left hand mutually perpendicular to each other. If the forefinger points in the direction of the magnetic field, central finger in the direction of current, then the thumb gives the direction of force on the charged particle.

(*ii*) ***Right hand palm rule.*** Open the right hand and place it so that tips of the fingers point in the direction of the field $\vec{B}$ and thumb in the direction of velocity $\vec{v}$ of positive charge, then the palm faces towards the force $\vec{F}$.

12. Definition of magnetic field. The magnetic field at a point may be defined as the force acting on a unit charge moving with a unit velocity at right angles to the direction of the field.

13. SI unit of magnetic field is tesla. One tesla is that magnetic field in which a charge of 1 coulomb moving with a speed of 1 ms^{-1} at right angles to the field experiences a force of 1 newton.

1 tesla (T) = 1 N $A^{-1} m^{-1}$,

1 gauss (G) = 10^{-4} T.

14. Lorentz force. The total force, called Lorentz force, acting on a charge q moving with velocity $\vec{v}$ in an electric field $\vec{E}$ and magnetic field $\vec{B}$ is

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

15. Motion of charge inside an electric field. If a potential difference V is applied between two parallel plates separated by distance d, then electric field set up between the plates is

$$E = \frac{V}{d}$$

The charge q of mass m experiences the electric force, $F_e = qE$

Acceleration produced in the charge, $a = \frac{qE}{m}$

The moving charge follows a parabolic path inside the electric field.

16. Motion of charge inside a magnetic field.
(*i*) When $\vec{v} \perp \vec{B}$, the magnetic force on the charge makes it move along a circular path of radius,

$$r = \frac{mv}{qB}$$

(*ii*) When $\vec{v}$ makes angle θ with $\vec{B}$, the perpendicular component : $v_{\perp} = v \sin\theta$ of the initial velocity makes the charge move along a circular path of radius,

$$r = \frac{mv \sin\theta}{qB}$$

The parallel component : $v_{\parallel} = v \cos\theta$ of the initial velocity makes it move along the direction of the magnetic field. The resultant of the two components makes the charge move along a helical path of pitch,

$$h = \frac{2\pi mv \cos\theta}{qB}.$$

17. Cyclotron. It is a device used to accelerate charged particles like protons, deutrons, α-particles, etc. to very high energies. Here charged particles move along a spiral path under the action of a perpendicular magnetic field and gain energy as they cross an alternating electric field again and again.

18. Cyclotron frequency. In a cyclotron, the frequency of the applied alternating electric field is equal to the frequency of revolution of the charged particle. This frequency is called *cyclotron frequency*.

It is given by, $f_c = \frac{qB}{2\pi m}$

where m is the mass and q is the charge of the positive ion. The cyclotron frequency is independent of both the velocity of the particle and the radius of its orbit.

19. Maximum energy gained by positive ions. If v_0 and r_0 are the maximum velocity and maximum radius of the circular path of the positive ions in a cyclotron, then

$$\frac{mv_0^2}{r_0} = qv_0 B \quad \text{or} \quad v_0 = \frac{qBr_0}{m}$$

$\therefore$ Maximum kinetic energy $= \frac{1}{2} mv_0^2 = \frac{q^2 B^2 r_0^2}{2m}$

If V is the accelerating potential of the high frequency oscillator and the charged particle completes n revolutions before leaving the dees, then

Maximum kinetic energy $= 2nqV$.

20. Force on a current carrying conductor in a magnetic field. A conductor of length l carrying current I held in a magnetic field $\vec{B}$ at an angle θ with it, experiences a force given by

$$F = I\,lB \sin\theta$$

In vector notation, $\vec{F} = I(\vec{l} \times \vec{B})$

The direction of $\vec{F}$ is perpendicular to both $\vec{l}$ and $\vec{B}$ and is given by Fleming's left hand rule. The force is maximum when $\theta = 90°$ and zero when $\theta = 0°$ or $180°$.

$$F_{max} = I\,lB$$

21. Force between two parallel infinitely long current carrying conductors. The force per unit length between two long parallel conductors carrying currents I_1 and I_2 and separated by distance r is given by

$$f = \frac{\mu_0}{2\pi} \cdot \frac{I_1 I_2}{r}$$

The force is attractive when the currents are in the same direction and repulsive when the currents are in opposite directions.

22. SI unit of current is ampere. One ampere is that strength of current which, when flowing in two parallel infinitely long conductors of negligible cross-section placed in vacuum at a distance of 1 m from each other, produces between them a force of 2×10^{-7} newton per metre length.

23. Force between two current elements. The force between two parallel current elements dl_1 and dl_2 carrying currents I_1 and I_2 and separated by distance r much greater than their length is given by

$$dF = \frac{\mu_0 I_1 I_2}{4\pi} \cdot \frac{dl_1\, dl_2}{r^2}$$

24. Relative sizes of electric and magnetic fields. Under similar conditions, the magnetic forces are much smaller than the electric forces.

$$\frac{F_m}{F_e} = v_1 v_2 (\mu_0 \varepsilon_0) = \frac{v_1 v_2}{c^2}$$

$$= \frac{10^{-5} \times 10^{-5}}{(3 \times 10^8)^2} \simeq 10^{-27}$$

Here v_1 and v_2 are drift speeds of the electrons in the two conductors.

25. Torque on current carrying coil in a magnetic field. A rectangular coil of area A, carrying current I and capable of rotation about an axis perpendicular to the field $\vec{B}$ experiences a torque,

$$\tau = NIBA \sin\theta = m B \sin\theta$$

where N = number of turns in the coil, $m = NIA$ = magnetic dipole moment, θ = angle which the normal to the plane of the coil makes with the field $\vec{B}$.

In vector rotation, $\vec{\tau} = \vec{m} \times \vec{B}$

Torque is minimum when the plane of the coil is perpendicular to the magnetic field ($\theta = 0°$).

Torque is maximum when the plane of the coil is parallel to the magnetic field ($\theta = 90°$).

$$\tau_{max} = NIBA.$$

26. Moving coil galvanometer. It is a device used to detect current in a circuit. It is based on the principle that a current carrying coil placed in a magnetic field experiences a current dependent torque, which tends to rotate the coil and produces angular deflection. It consists of a coil of wire of area A and N turns carrying current I to be measured. It is suspended in a radial magnetic field so that its plane always remains parallel to $\vec{B}$ ($\sin\theta = 1$) by a suspension fibre of torsion constant k. In equilibrium position,

Restoring torque = Deflecting torque

or $\quad k\alpha = NIBA \quad$ or $\quad \alpha = \frac{NBA}{k} \cdot I$

i.e., Deflection of coil $\propto$ Current in the coil.

27. Figure of merit of a galvanometer. It is the current which produces a deflection of one scale division in the galvanometer. It is given by

$$G = \frac{I}{\alpha} = \frac{k}{NBA}.$$

28. Current sensitivity of a galvanometer. It is the deflection produced in a galvanometer when a unit current flows through it.

$$\text{Current sensitivity} = \frac{\alpha}{I} = \frac{NBA}{k}.$$

29. Voltage sensitivity of a galvanometer. It is the deflection produced in a galvanometer when a unit potential difference is applied across its ends.

$$\text{Voltage sensitivity} = \frac{\alpha}{V} = \frac{\alpha}{IR} = \frac{NBA}{kR}$$

30. Conversion of a galvanometer into an ammeter. An ammeter is an instrument used to measure electric current in a circuit. A galvanometer of resistance G can be converted into an ammeter of range $0 - I$ by connecting a small resistance S in parallel with it. The value of small resistance, called shunt, is given by

$$S = \frac{I_g}{I - I_g} \times G$$

where I_g is the current with which galvanometer gives full-scale deflection.

Total resistance of an ammeter,

$$R_A = \frac{GS}{G+S}$$

An ammeter is a low resistance device and is connected in series in a circuit.

31. Conversion of galvanometer into a voltmeter. A voltmeter is used to measure potential difference between any two points of a circuit. A galvanometer of resistance G can be converted into a voltmeter of range $0 - V$ by connecting a large resistance R in series with it. The value of R is given by

$$R = \frac{V}{I_g} - G$$

Total resistance of a voltmeter,

$$R_V = G + R$$

A voltmeter is a high resistance device and is always connected parallel to the conductor across which p.d. is to be measured.